CQE Academy

CQE EXAM

The Full Equation List!

ALL 165 Equations You'll need to CRUSH the CQE Exam

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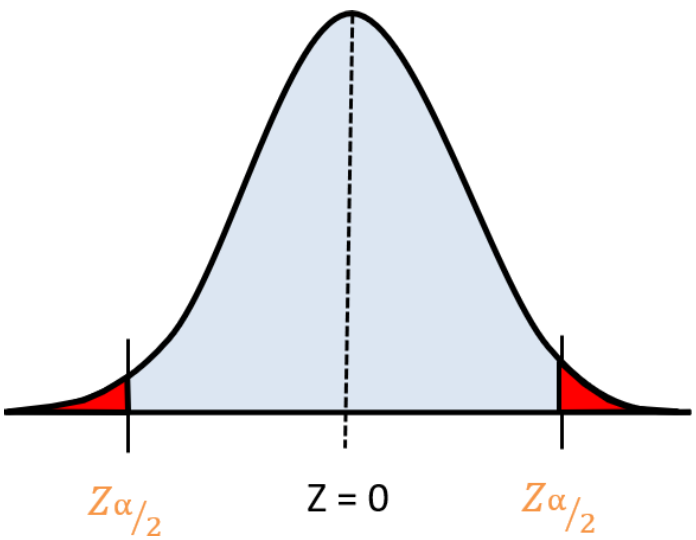
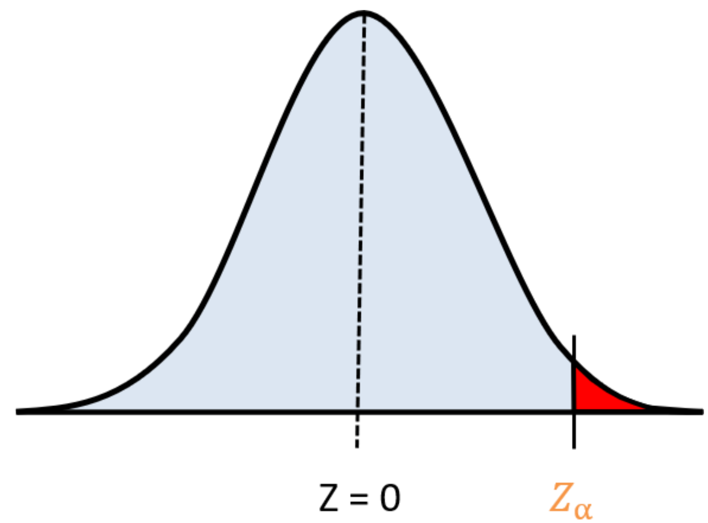
Feature	Description	Definition	Calculation for **Sample** Data	Calculation for **Population** Data
Central Tendency	Mean	The arithmetic average of all observations	$\bar{x} = \frac{\sum x}{n}$	$\mu = \frac{\sum X}{N}$
	Median	The middle value (mid-point) of a data set 3, 1, 2, 2, 4, 5, 2, 6, 7 -> **3**	If the data set has an even number of points, take the average of the 2 middle points.	
	Mode	The most frequently occurring value in a data set 3, 1, 2, 2, 4, 5, 2, 6, 7 -> **2**	Count the frequency of each value in the data set.	
Dispersion	Variance	The average squared difference of each individual value from the mean.	$s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$	$\sigma^2 = \frac{\sum(x-\bar{\mu})^2}{N}$
	Standard Deviation	The square root of the variance.	$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$	$\sigma = \sqrt{\frac{\sum(x-\bar{\mu})^2}{N}}$
	Range	The smallest interval containing all of the data. Calculated as the difference between the Max and Min values of a data set.	$Range = R = Max(x) - Min(x)$	

Probability Concept	Equation
The Union ("OR")	$The\ Probability\ of\ A\ or\ B = P(A\ U\ B) = P(A) + P(B) - P(A \cap B)$
The Intersection ("AND")	$The\ Probability\ of\ A\ AND\ B = P(A \cap B)$ $For\ Mutually\ Exclusive\ Events:\ P(A \cap B) = 0$
The Complement	$The\ Probability\ of\ A^{\circ} = P(A^{\circ}) = 1 - P(A)$
Conditional Probability	$Probabiliy\ of\ A\ given\ B = P(A\,\vert B) = \frac{P(A \cap B)}{P(B)} = \frac{The\ Intersection\ of\ A\ \&\ B}{The\ Probability\ of\ B}$
Probability for Independent Events	$For\ independent\ Events: P(A \cap B \cap C) = P(A) * P(B) * P(C)$
Addition Rule ("OR") (Non-Mutually Exclusive Events)	$P(A\ U\ B) = P(A) + P(B) - P(A \cap B)$
Addition Rule (Mutually Exclusive Events)	$P(A\ U\ B) = P(A) + P(B)$
Multiplication Rule (Independent Events)	$For\ Independent\ Events:\ P(A\ and\ B) = P(A \cap B) = P(A) * P(B)$
Multiplication Rule (Dependent Events)	$For\ Dependent\ Events:\ P(A\ and\ B) = P(A \cap B) = P(A\vert B) * P(B)$

Continuous Data Distribution

Distribution	Cumulative Probability	Central Tendency	Dispersion	Parameters
The Normal Distribution	$Z = \frac{X - \mu}{\sigma}$	$\mu = \frac{\sum X}{N}$	$\sigma = standard\ deviation$	**μ** is the mean value **σ** is the standard deviation
The Uniform Distribution	$P(X_1 < x < X_2) = \frac{(X_1 - X_2)}{(b - a)}$	$\mu = \frac{a + b}{2}$	$\sigma^2 = \frac{(b - a)^2}{12}$	***a*** is the minimum value in the distribution ***b*** is the maximum value in the distribution
The Exponential Distribution	$R(t) = \mathrm{e}^{-\lambda t}$	$\theta = \frac{1}{\lambda}$		$\boldsymbol{\lambda} = Failure\ Rate$
The Weibull Distribution	$R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$			**β(Beta)** - the Weibull **Shape** or Slope Parameter **θ(Theta)** - the Weibull **Scale** Parameter **Δ(Delta)** - The Weibull **Location** Parameter (Not shown in equation)





Significance Level (α)	Confidence Level (100 – α)	Two-tailed Sign. Level $(\alpha/2)$	$Z_{\alpha/2}$
0.01	99%	0.005	2.575
0.05	95%	0.025	1.960
0.10	90%	0.05	1.645

Significance Level (α)	Confidence Level (100 – α)	Z_{α}
0.01	99%	2.33
0.05	95%	1.65
0.10	90%	1.28

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Discrete Data Distribution

Distribution	Mean	Standard Deviation	Probability	Parameters	
The Binomial Distribution	$\mu = n*p$	$\sigma = \sqrt{n * p(1-p)}$	$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	**n** is the sample size **p** is the probability of success	$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$
The Poisson Distribution	$\mu = \lambda$	$\sigma = \sqrt{\lambda}$	$P(X = x) = \frac{e^{-\lambda} * \lambda^x}{x!}$	**e** = Euler's Number = 2.71828.... **λ** = mean number of occurrences	
The Hypergeometric Distributions	NA	NA	$f(x) = \frac{\binom{A}{x} * \binom{N-A}{n-x}}{\binom{N}{n}}$	**N** is the population quantity **n** is the sample quantity **A** is the number of "Successes" in the population	$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$

Sampling Distribution

Distribution	Test Statistic	Parameters	Usage
The F Distribution	$F = \frac{(S_1)^2}{(S_2)^2}$	$(S_1)^2$ is the first sample variance $(S_2)^2$ is the second sample variance	Hypothesis Testing, ANOVA Analysis
Student T Distribution	$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$	$\bar{x}$ is the sample mean, **μ** is the population mean **s** is the sample standard deviation **n** is the sample size	Confidence Intervals, Hypothesis Testing
The Chi-Squared Distribution	$X^2 = \frac{(N-1)s^2}{\sigma^2}$ or $X^2 = \sum_{i}^{i=k} \frac{(O_i - E_i)^2}{E_i}$	**N** is the sample size, s^2 is the sample variance σ^2 is the population variance O_i = Observed Value, E_i = Expected Value	Confidence Intervals, Hypothesis Testing, Contingency Tables, Goodness of Fit Testing

Critical Values of the Student T's Distribution at Various Degrees of Freedom and Alpha Levels (α)

df (v)	α = 0.1	α = 0.05	α = 0.025	α = 0.01	α = 0.005	α = 0.001
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144

df (v)	α = 0.1	α = 0.05	α = 0.025	α = 0.01	α = 0.005	α = 0.001
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552

Confidence Intervals

Parameter of Interest	Equation	Parameters	Use When
The Population Mean (μ)	$\bar{x} \pm Z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}$	$\bar{x}$ is the sample mean, **σ** is the population standard deviation, **n** is the sample size, $Z_{\frac{\alpha}{2}}$ is the z-statistic associated with the confidence level	The population variance is known and the sample size (n) is greater than 30
The Population Mean (μ)	$\bar{x} \pm t_{\frac{\alpha}{2}} * \frac{s}{\sqrt{n}}$	$\bar{x}$ is the sample mean, **s** is the sample standard deviation, **n** is the sample size, $t_{\frac{\alpha}{2}}$ is the t-statistic associated with the confidence level	The population variance is unknown or the sample size (n) is less than 30
The Population Variance	$\frac{(n-1)s^2}{X^2_{1-\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{X^2_{\alpha/2}}$	**n** is the sample size, s^2 is the sample variance, $X^2_{1-\alpha/2}$ and $X^2_{\alpha/2}$ are the critical chi-squared values associated with the confidence level	Creating a confidence interval for the population variance when the sample variance is known.
The Population Standard Deviation	$\sqrt{\frac{(n-1)s^2}{X^2_{1-\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)s^2}{X^2_{\alpha/2}}}$	**n** is the sample size, s^2 is the sample variance, $X^2_{1-\alpha/2}$ and $X^2_{\alpha/2}$ are the critical chi-squared values associated with the confidence level	The square root of variance.
The Population Proportion	$p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p*(1-p)}{n}}$	**p** is the sample proportion, **n** is the sample size, and $Z_{\frac{\alpha}{2}}$ is the z-statistic associated with the confidence level	Creating a confidence interval for the population proportion when the sample proportion is known.

Hypothesis Testing

Parameter of Interest	Equation	Parameters	Use When
Population Mean	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$	$\bar{x}$ is the sample mean, **μ** is the population mean, **σ** is the population standard deviation**,** **n** is the sample size	The population variance is known and the sample size (n) is greater than 30
Population Mean	$t - statistic = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$	$\bar{x}$ is the sample mean, **μ** is the population mean, **s** is the sample standard deviation**;** **n** is the sample size	The population variance is unknown or the sample size (n) is less than 30
Population Variance	$X^2 = \frac{(n-1)s^2}{\sigma^2}$	**n** is the sample size, S^2 is the sample variance, σ^2 is the population variance	Comparing a sample variance against a population variance
Population Variance	$F = \frac{s_1^2}{s_2^2}$	$(S_1)^2$ is the first sample variance $(S_2)^2$ is the second sample variance	Comparing two population variances against each other
Population Proportions	$Z_O = \frac{\hat{p} - p_O}{\sqrt{\frac{p_O(1-p_O)}{n}}}$	p_O = the hypothesized population proportion, $\hat{p}$ is the sample proportion, **n** is the sample size,	Comparing a sample proportion against a population proportion

Goodness of Fit Testing

Parameter of Interest	Equation	Parameters	Use When
Chi-Squared	$X^2 = \sum_{i}^{i=k} \frac{(O_i - E_i)^2}{E_i}$	**O_i** = Observed Value from sample data, **E_i** = Expected Value from the assumed population.	Comparing sample data (observed values) against the expected values from an assumed population.

Contingency Tables

Parameter of Interest	Equation	Parameters	Use When
Chi-Squared	$X^2 = \sum_{i}^{i=k} \frac{(O_i - E_i)^2}{E_i}$	$\boldsymbol{O_i}$ = Observed Value from the sample data $\boldsymbol{E_i} = \frac{Row_{total} * Column_{total}}{Grand\ Total} = \frac{R * C}{N}$	Comparing observed values from a contingency table against the expected values (**E_i**) to determine if the two factors are independent. $df_{contingency} = (r-1)(c-1)$

ANOVA

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	Sum of Squares of the Treatment **(SS_t)**	DF of the Treatment **(DF_t)**	Mean Squares of the Treatment **(MST)**	**= MST / MSE**
Error (Within)	Sum of Squares of the Error **(SS_e)**	DF of the Error **(DF_e)**	Mean Squares of the Error **(MSE)**	
Total	Total Sum of Squares **(SS_{total})**	Total DF **(DF_{Total})**		

$$SS_{total} = \sum (X_i - GM)^2 = SS_e + SS_t$$

$$SS_e = \sum (X_i - \bar{X})^2$$

$$SS_t = \sum n(\bar{X}_i - GM)^2 \quad \text{where GM = Grand Mean}$$

$$Mean\ Square\ of\ the\ Treatment = \frac{SS_{Treatment}}{DF_{Treatment}}$$

$$Mean\ Square\ of\ the\ Error = \frac{SS_{error}}{DF_{error}}$$

$$F - statistic = \frac{MST}{MSE}$$

$DF_{total} = DF_{error} + DF_{treatment} = N - 1$	N = *total observations*
$DF_{treatment} = a - 1$	a = *Number of treatments*
$DF_{error} = (N-1) - (a-1) = a(n-1)$	n = *samples per treatment group*

Relationships Between Variables

$$\hat{y} = \beta_1 x + \beta_0 + \varepsilon$$

$$Slope = \beta_1 = \frac{S_{xy}}{S_{x^2}}$$

$$Y - Intercept = \beta_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\varepsilon = y - \hat{y}$$

$$S_{xy} = \sum (X_i * Y_i) - \frac{(\sum X_i)(\sum Y_i)}{n} \qquad S_{x^2} = \sum X_i^2 - \frac{(\sum X_i)^2}{n}$$

$$\bar{X} = \frac{\sum X_i}{n} \qquad \bar{Y} = \frac{\sum Y_i}{n}$$

$$S_{y^2} = \sum Y_i^2 - \frac{(\sum Y_i)^2}{n} \qquad S_{x^2} = \sum X_i^2 - \frac{(\sum X_i)^2}{n}$$

$$SSE = \sum (y - \hat{y})^2 \quad Or \quad SSE = S_{yy} - \beta_1 S_{xy}$$

$$Sample\ Variance\ of\ the\ Error: s_e^2 = \frac{SSE}{n-2}$$

Hypothesis Testing for Linear Relationship

$H_0: \beta_1 = 0$ & $H_A: \beta_1 \neq 0$

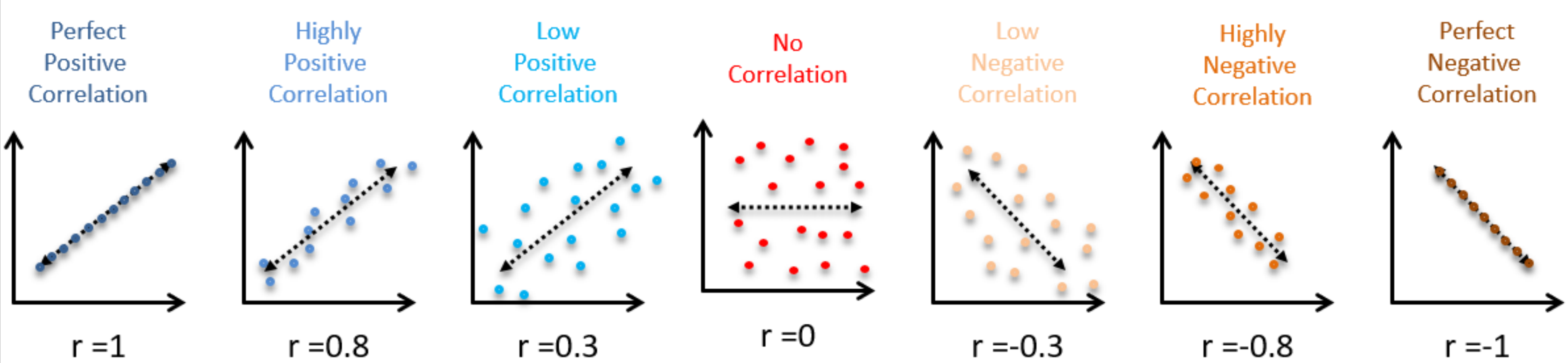
$$t = \frac{\hat{\beta}_1 - \beta_1}{S_e / \sqrt{S_{xx}}}$$

$$degrees\ of\ Freedom\ d.f. = n - 2$$

Correlation Coefficients

Parameter	Equation	Description
r	$r_{xy} = \frac{S_{xy}}{\sqrt{S_{xx}} * \sqrt{S_{yy}}}$	The **Pearson Correlation Coefficient, r**, measures the strength of the linear relationship between two variables which means that a change in one variable (the independent variable) also implies a change in the other (dependent variable).
R^2	$R^2 = r_{xy}^2$	The **coefficient of determination**, **R^2**, reflects the proportion of the total variability in the Y variable that can be explained by the regression line, and thus it also reflects the adequacy of your model for a particular sample of data.

Scatter Plots & Correlation Examples



Statistical Process Control

The X-BAR AND R CHART

$$\textbf{The Grand Mean} = \bar{\bar{X}} = \frac{\sum \bar{X}_i}{k} = \frac{Sum\ of\ Subgroup\ Average\ Values}{\#\ of\ Subgroups}$$

$$\textbf{X-bar Control Limits}: UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} \qquad LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

$$\textbf{The Average Range} = \bar{R} = \frac{\sum R_i}{k} = \frac{Sum\ of\ Subgroup\ Ranges}{\#\ of\ Subgroups}$$

$$\textbf{The R Chart Control Limits}: \quad UCL_R = D_4\bar{R} \qquad LCL_R = D_3\bar{R}$$

$$\textbf{Population Standard Deviation} = \hat{\sigma} = \frac{\bar{R}}{d_2}$$

The X-BAR AND S CHART

$$\textbf{The Grand Mean}: \bar{\bar{X}} = \frac{\sum \bar{X}_i}{k} = \frac{Sum\ of\ Subgroup\ Average\ Values}{\#\ of\ Subgroups}$$

$$\textbf{X-bar Chart Control Limits}: UCL_{\bar{X}} = \bar{\bar{X}} + A_3\bar{s} \qquad LCL_{\bar{X}} = \bar{\bar{X}} - A_3\bar{s}$$

$$\textbf{The Average Sample Standard Deviation}: \bar{s} = \frac{\sum s_i}{k} = \frac{Sum\ of\ Subgroup\ St.Dev.}{\#\ of\ Subgroups}$$

$$\textbf{The S Chart Control Limits}: \quad UCL_s = B_4\bar{s} \qquad LCL_s = B_3\bar{s}$$

$$\textbf{Population Standard Deviation} = \hat{\sigma} = \frac{\bar{s}}{C_4}$$

X-Bar and R Chart

Subgroup Sample Size	X-Bar Factor	Range Factors		Variance Factor
n	A_2	D_3	D_4	d_2
2	1.880	-	3.267	1.128
3	1.023	-	2.575	1.693
4	0.729	-	2.282	2.059
5	0.577	-	2.115	2.326
6	0.483	-	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
15	0.223	0.347	1.653	3.472

X-Bar and S Chart

Subgroup Sample Size	X-Bar Factor	Standard Deviation Factors		Variance Factor
n	A_3	B_3	B_4	c_4
2	2.659	-	3.267	0.7979
3	1.954	-	2.568	0.8862
4	1.628	-	2.266	0.9213
5	1.427	-	2.089	0.9400
6	1.287	0.030	1.970	0.9515
7	1.182	0.118	1.882	0.9594
8	1.099	0.185	1.815	0.9650
9	1.032	0.239	1.761	0.9693
10	0.975	0.284	1.716	0.9727
15	0.789	0.428	1.572	0.9823
20	0.680	0.510	1.490	0.9869
25	0.606	0.565	1.435	0.9896

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The I-MR Chart

$$\textit{Individual Chart Centerline} = \bar{X} = \frac{\sum X_i}{k} = \frac{\textit{Sum of Individual Values}}{\textit{\# of Individual Values}}$$

$$\textit{Individual Chart Control Limits}: \quad UCL_I = \bar{X} + E_2\overline{MR} \qquad LCL_I = \bar{X} - E_2\overline{MR}$$

$$\textit{Moving Range } (\overline{MR}) \textit{ Centerline} = \frac{\sum MR_i}{k-1} = \frac{\textit{Sum of Moving Ranges}}{\textit{\# of MR's}}$$

$$\textit{Moving Range Control Limits}: \quad UCL_{MR} = D_4\overline{MR} \qquad LCL_{MR} = D_3\overline{MR}$$

I-MR Chart				
Subgroup Sample Size	Individual Factor	Moving Range Factors		Variance Factor
n	E_2	D_3	D_4	d_2
2	2.660	-	3.267	1.128
3	1.772	-	2.575	1.693
4	1.457	-	2.282	2.059
5	1.290	-	2.115	2.326
6	1.184	-	2.004	2.534
7	1.109	0.076	1.924	2.704
8	1.054	0.136	1.864	2.847
9	1.010	0.184	1.816	2.970
10	0.975	0.223	1.777	3.078

Attribute Data Control Charts

np & p charts trend defectives and are based on the **Binomial distribution** which operates under the assumption that every unit inspected can only be counted as "bad" one time.

p-Chart trends *defectives* with a *Variable* Sample Size

np-Chart trends *defectives* with a *Constant* Sample Size

u & c Charts utilize the **Poisson distribution** as they trend the number of defects where it is possible for each item inspected to contain multiple defects.

u-Chart trends *defects* with a *Variable* Sample Size

c-Chart trends *defects* with a *Constant* Sample Size

		Sample Size	
		Constant	**Variable**
Type	**Defect**	c Chart	u Chart
	Defectives	np Chart	p Chart

Attribute Data Control Charts

The p Chart

$$\bar{p} = Centerline = \frac{\sum np}{\sum n} = \frac{Sum\ of\ All\ Defectives}{Sum\ of\ Subgroup\ Quantity}$$

$$UCL_{\bar{p}} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} \qquad LCL_{\bar{p}} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$

$$\bar{n} = Average\ Sample\ Size = \frac{\sum n}{k} = \frac{\sum subgroup\ quantity}{\#\ of\ subgroups}$$

The np Chart

$$n\bar{p}\ Centerline = \frac{\sum np}{k} = \frac{Sum\ of\ All\ Defectives}{\#\ of\ subgroups}$$

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \qquad LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\bar{p} = \%\ Defective = \frac{\sum np}{\sum n} = \frac{Sum\ of\ All\ Defectives}{Sum\ of\ Subgroup\ Quantity}$$

The u Chart

$$\bar{u} = Centerline = \frac{\sum c}{\sum n} = \frac{Sum\ of\ All\ Defects}{Sum\ of\ units\ inspected}$$

$$\bar{n} = Average\ \ of\ Samples\ per\ Subgroup = \frac{\sum n}{k} = \frac{\text{Sum of units inspected}}{\text{Number of subgroups}}$$

$$UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}} \qquad LCL_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

The c Chart

$$\bar{c} = Centerline = \frac{\sum c}{k} = \frac{Sum\ of\ All\ Defects}{\#\ of\ Subgroups}$$

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} \qquad UCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

		Sample Size	
		Constant	**Variable**
Type	**Defect**	c Chart	u Chart
	Defectives	np Chart	p Chart

Process Capability

$$C_p = \frac{USL - LSL}{6s_{c_p}} \qquad C_r = \frac{1}{C_p} \qquad C_{pk} = Min(C_{p,Lower}, C_{p,Upper}) = Min\left(\frac{USL - \bar{x}}{3s_{c_p}}, \frac{\bar{x} - LSL}{3s_{c_p}}\right)$$

$$s_{c_p} = \frac{\bar{R}}{d_2}\ (if\ using\ an\ \bar{X} - R\ Chart) \quad or \quad s_{c_p} = \frac{\bar{s}}{c_4}\ (if\ using\ an\ \bar{X} - S\ Chart)$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{s^2 + (\bar{x} - T)^2}}$$

Process Performance Indices

$$P_p = \frac{USL - LSL}{6s_{p_p}} \qquad P_{pk} = Min\left(\frac{USL - \bar{x}}{3s_{p_p}}, \frac{\bar{x} - LSL}{3s_{p_p}}\right) \qquad s_{p_p} = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

Design of Experiments

Treatments in a DOE

$$\textbf{Full}\ Factorial\ Design: Number\ of\ Treatments = Levels^{Factors} = L^F = 2^F$$

$$\textbf{Half}\ Factorial\ Design: Number\ of\ Treatments = \frac{Levels^{Factors}}{2} = \frac{L^F}{2} = \frac{2^F}{2} = 2^{F-1}$$

$$\textbf{Quarter}\ Factorial\ Design: Number\ of\ Treatments = \frac{Levels^{Factors}}{4} = \frac{L^F}{4} = \frac{2^F}{2^2} = 2^{F-2}$$

# of Factors	Full Factorial Exp.	Half Factorial Exp.	Quarter Factorial Exp.
2	4	2	1
3	8	4	2
4	16	8	4
5	32	16	8
6	64	32	16
7	128	64	32
8	256	128	64
9	512	256	128
10	1024	512	256

Calculating Effects from a DOE

$$\textbf{Estimated Effect} = Average\ at\ High - Average\ at\ Low$$

Reliability Equations

The Exponential Distribution

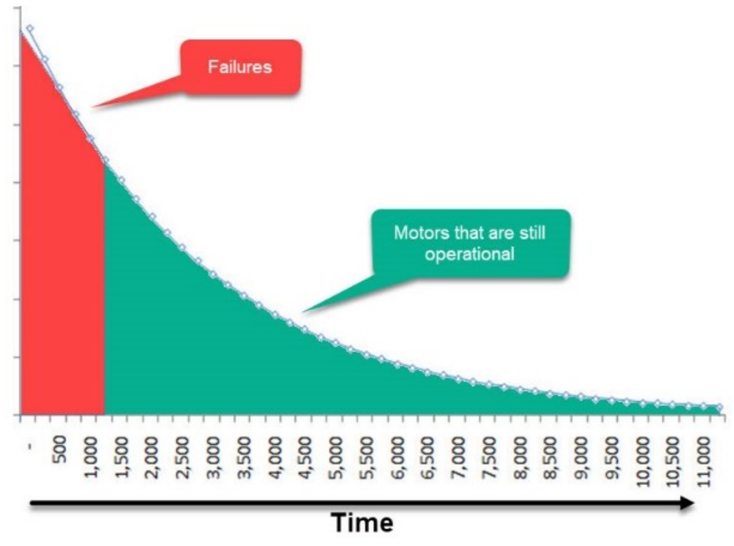
$$\textbf{Reliability}: \quad R(t) = e^{-\lambda t} = e^{\frac{-t}{\theta}} \qquad \textbf{Failures}: \quad F(t) = 1 - e^{-\lambda t} \qquad \textbf{PDF}: \quad f(t) = \lambda e^{-\lambda t}$$

$$\textbf{Failure Rate} = \boldsymbol{\lambda} = \frac{1}{\theta} = \frac{1}{MTTF} = \frac{Number\ of\ Failures}{Operating\ Time(Cycles)}$$

$$\boldsymbol{\theta} = MTBF\ (or\ MTTF) = \frac{1}{\lambda} = \frac{Operating\ Time\ (Cycles)}{Number\ of\ Failures}$$

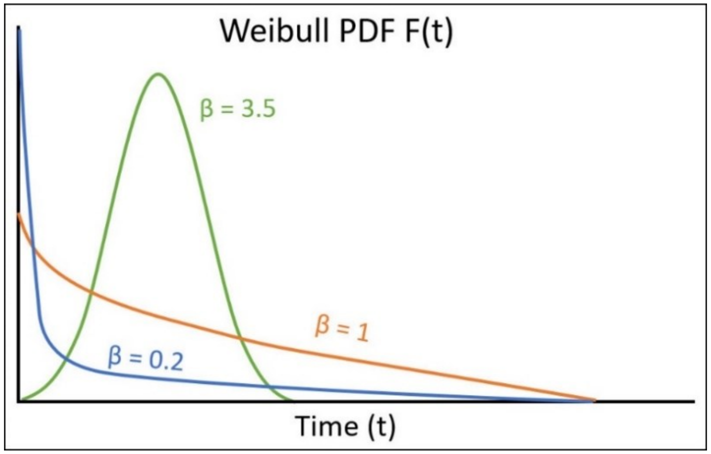
Mean Time to Failure (**MTTF**) is only used when an item is not repairable.

Mean Time Between Failures (**MTBF**) is used if a product or asset is repairable.



Weibull Distribution Equations

$\textbf{PDF}: \quad f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}}$	$\textbf{Reliability}: \quad R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$
$\textbf{CDF}: \quad F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}}$	$\textbf{Failure Rate}: \quad h(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$



Weibull Parameters

β(Beta) - the Weibull Shape or Slope Parameter
θ(Theta) - the Weibull Scale Parameter
Δ(Delta) - The Weibull Location Parameter (Not shown above)

Maintainability and Availability

$$\textbf{MTTR} = \frac{Total\ Time\ Spent\ in\ Maintenance}{Total\ Number\ of\ Maintenance\ Activities\ conducted} = Maintainability$$

$$\textbf{Availability} = \frac{MTBF}{MTBF + MTTR}$$

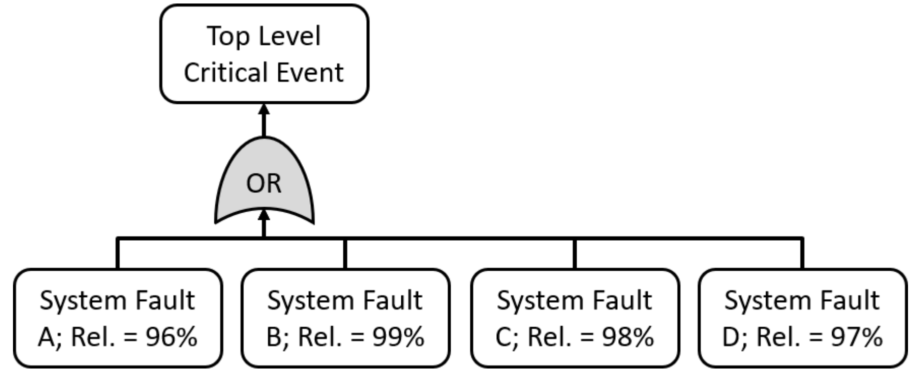
Reliability and Fault Tree Equations

Series System Reliability

$$\textit{Series System Reliability} = R_{system} = R_1 \times R_2 \times R_3 \times R_4 \times \ldots \ldots R_n$$

Fault Tree Reliability (OR GATE)

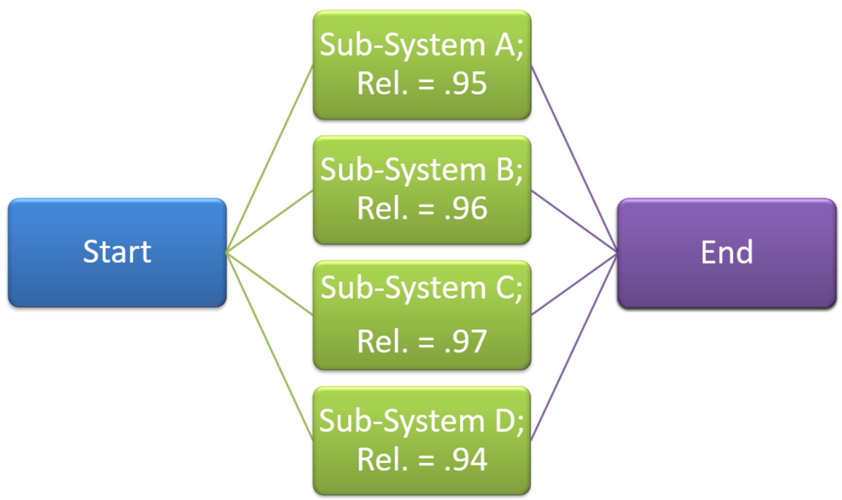
$$\textit{Reliability of OR Gate}: \ R_{OR\ Gate} = R_1 \times R_2 \times R_3 \times R_4 \times \ldots \ldots . R_n$$



Parallel System Reliability

$$\textit{Parallel System Reliability} = 1 - (U_1 \times U_2 \times U_3 \times U_4 \times \ldots . . U_n)$$

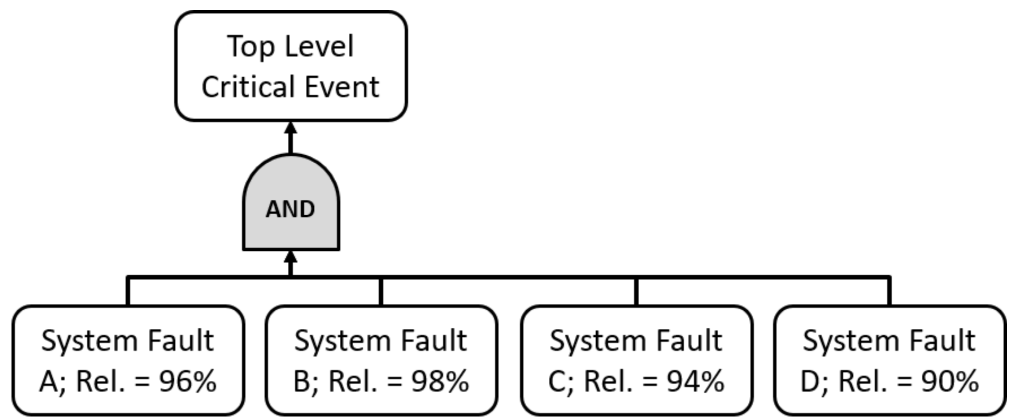
$$\textit{Where } U_1 = 1 - R_1$$



Fault Tree Reliability (AND GATE)

$$\textit{AND GATE System Reliability} = 1 - (U_1 \times U_2 \times U_3 \times U_4 \times \ldots . . U_n)$$

$$\textit{Where } U_1 = 1 - R_1$$



Gauge R&R (Measurement System Analysis)

The Average and Range Method

$$Equipment\ Variation\ (EV) = \sigma_{Repeatability} = \frac{\bar{\bar{R}}}{d_2}$$

$$Appraiser\ Variation\ (AV) = \sigma_{Reproducibility} = \sqrt{\left(\frac{R_o}{d_2}\right)^2 - \frac{(EV)^2}{n * r}}$$

$$Part\ to\ Part\ Variation\ (PV) = \sigma_p = \frac{R_p}{d_2}$$

$$Measurement\ System\ Variation\ (GR\&R) = \sigma_{GRR} = \sqrt{(EV)^2 + (AV)^2}$$

$$Total\ Variation = \sqrt{(GR\&R)^2 + (PV)^2}$$

The Range Method

$$Measurement\ System\ Variation\ (GRR) = \sigma_{GRR} = \frac{\bar{R}}{d_2^*}$$

The ANOVA Method

$$Repeatability - Equipment\ Variation\ (EV):\ \sigma^2_{Repeatabilitiy} = MS_E$$

$$Repeatability\ -\ Appraiser\ Variation\ (AV):\ \sigma^2_{Reproducibility} = \frac{MS_O - MS_{PxO}}{P * R}$$

$$Appraiser - Part\ Interaction\ Variation\ (I_{PxO}):\ \sigma^2_{Interaction} = \frac{MS_{PxO} - MS_E}{R}$$

$$Part\ Variation\ (PV):\ \sigma^2_{Part} = \frac{MS_P - MS_{PxO}}{O * R}$$

$$Measurement\ System\ Variation\ (GRR) = \sigma_{GRR} = \sqrt{(EV)^2 + (AV)^2 + (I_{PxO})^2}$$

$$Total\ Variation = \sqrt{(GRR)^2 + (PV)^2}$$

The Precision/Tolerance (P/T) Ratio

$$\frac{P}{T}\ Ratio = \frac{Total\ Measurement\ System\ Variation}{Tolerance}$$

$$\frac{P}{T}\ Ratio = \frac{6 * \sigma_{GRR}}{Upper\ Spec. - Lower\ Spec.}$$

# Samples	Sub-group size (m)		
(n)	2	3	4
1	1.414	1.912	2.239
2	1.279	1.805	2.151
3	1.231	1.769	2.120
4	1.206	1.750	2.105
5	1.191	1.739	2.096
6	1.181	1.731	2.090
7	1.173	1.726	2.085
8	1.168	1.721	2.082
9	1.164	1.718	2.080
10	1.160	1.716	2.077
11	1.157	1.714	2.076
12	1.155	1.712	2.074
13	1.153	1.710	2.073
14	1.151	1.709	2.072
15	1.150	1.708	2.071
d_2^*	1.128	1.693	2.059

The Percent Variation Calculation

$$\textbf{Percent Variation} = \frac{Measurement\ System\ Variation}{Total\ Variation}$$

$$\sigma^2_{Total\ Variation} = \sigma^2_{GR\&R} + \sigma^2_{Part\ To\ Part\ Variation}$$

$$\textbf{Equipment Variation}\ (\textbf{EV} -\ Repeatability) = \frac{EV}{TV} * 100 = \frac{Equipment\ Variation\ (Repeatability)}{Total\ Variation}$$

$$\textbf{Appraiser Variation}\ (\textbf{AV} - Reproducibility) = \frac{AV}{TV} * 100 = \frac{Appraiser\ Variation\ (Reproducibility)}{Total\ Variation}$$

$$\textbf{Measurement System Variation} = \frac{GRR}{TV} * 100 = \frac{Gauge\ R\&R\ (Repeatability + Reproducibility)}{Total\ Variation}$$

$$\%\ \textbf{Part to Part Variation} = \frac{PV}{TV} * 100 = \frac{Part\ to\ Part\ Variation (PV)}{Total\ Variation}$$

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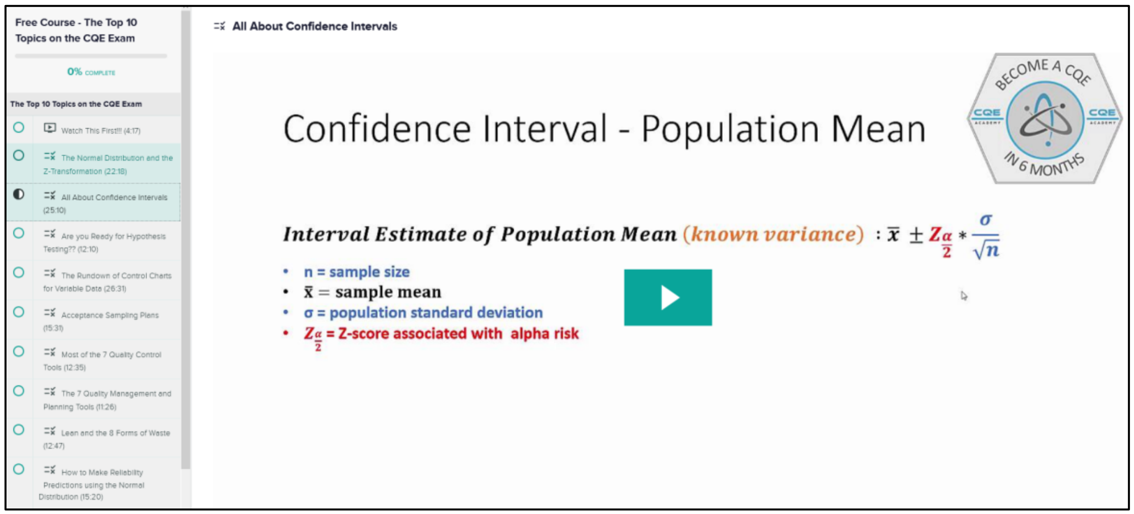

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-Andy

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