

CQE
Academy



CQE EXAM

The Full Equation List!

ALL 165 Equations
You'll need to CRUSH
the CQE Exam

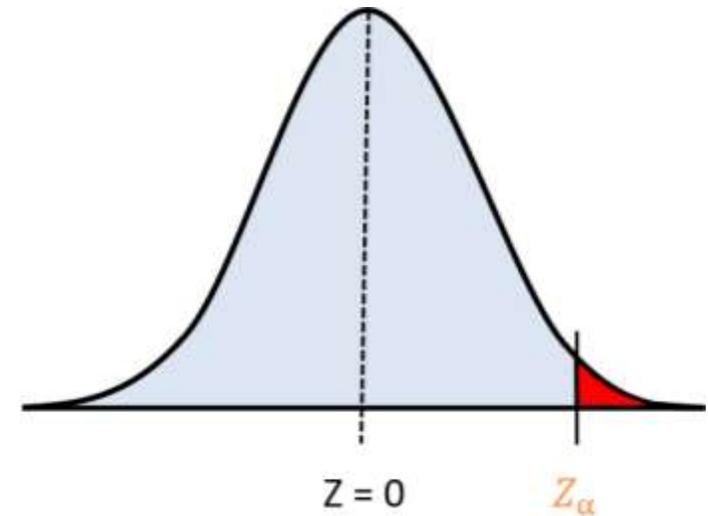
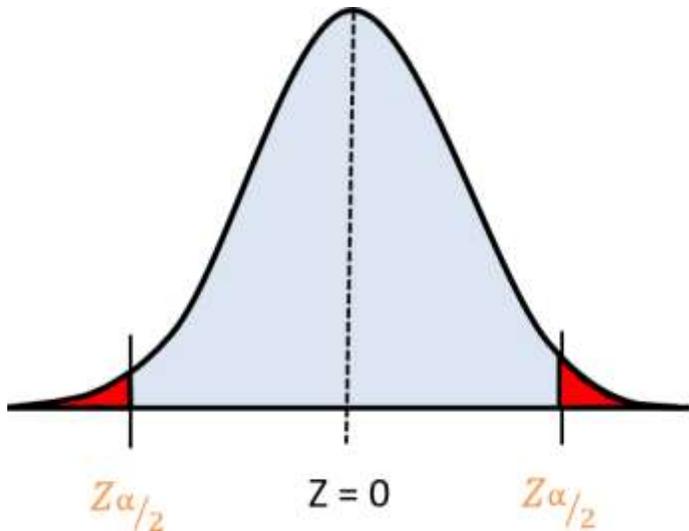
ANDY ROBERTSON

Feature	Description	Definition	Calculation for <u>Sample Data</u>	Calculation for <u>Population Data</u>
Central Tendency	Mean	The arithmetic average of all observations	$\bar{x} = \frac{\sum x}{n}$	$\mu = \frac{\sum X}{N}$
	Median	The middle value (mid-point) of a data set 3, 1, 2, 2, 4, 5, 2, 6, 7 → 3	If the data set has an even number of points, take the average of the 2 middle points.	
	Mode	The most frequently occurring value in a data set 3, 1, 2, 2, 4, 5, 2, 6, 7 → 2	Count the frequency of each value in the data set.	
Dispersion	Variance	The average squared difference of each individual value from the mean.	$s^2 = \frac{\sum(x - \bar{x})^2}{n - 1}$	$\sigma^2 = \frac{\sum(x - \bar{\mu})^2}{N}$
	Standard Deviation	The square root of the variance.	$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$	$\sigma = \sqrt{\frac{\sum(x - \bar{\mu})^2}{N}}$
	Range	The smallest interval containing all of the data. Calculated as the difference between the Max and Min values of a data set.	$Range = R = Max(x) - Min(x)$	

Probability Concept	Equation
The Union (“OR”)	$The\ Probability\ of\ A\ or\ B = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
The Intersection (“AND”)	$The\ Probability\ of\ A\ AND\ B = P(A \cap B)$ $For\ Mutually\ Exclusive\ Events: P(A \cap B) = 0$
The Complement	$The\ Probability\ of\ A^{\circ} = P(A^{\circ}) = 1 - P(A)$
Conditional Probability	$Probabiliy\ of\ A\ given\ B = P(A B) = \frac{P(A \cap B)}{P(B)} = \frac{The\ Intersection\ of\ A\ \&\ B}{The\ Probability\ of\ B}$
Probability for Independent Events	$For\ independent\ Events: P(A \cap B \cap C) = P(A) * P(B) * P(C)$
Addition Rule (“OR”) (Non-Mutually Exclusive Events)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Addition Rule (Mutually Exclusive Events)	$P(A \cup B) = P(A) + P(B)$
Multiplication Rule (Independent Events)	$For\ Independent\ Events: P(A\ and\ B) = P(A \cap B) = P(A) * P(B)$
Multiplication Rule (Dependent Events)	$For\ Dependent\ Events: P(A\ and\ B) = P(A \cap B) = P(A B) * P(B)$

Continuous Data Distribution

Distribution	Cumulative Probability	Central Tendency	Dispersion	Parameters
The Normal Distribution	$Z = \frac{X - \mu}{\sigma}$	$\mu = \frac{\sum X}{N}$	$\sigma = \text{standard deviation}$	μ is the mean value σ is the standard deviation
The Uniform Distribution	$P(X_1 < x < X_2) = \frac{(X_2 - X_1)}{(b - a)}$	$\mu = \frac{a + b}{2}$	$\sigma^2 = \frac{(b - a)^2}{12}$	a is the minimum value in the distribution b is the maximum value in the distribution
The Exponential Distribution	$R(t) = e^{-\lambda t}$	$\theta = \frac{1}{\lambda}$		$\lambda = \text{Failure Rate}$
The Weibull Distribution	$R(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}$			β (Beta) - the Weibull Shape or Slope Parameter θ (Theta) - the Weibull Scale Parameter Δ (Delta) - The Weibull Location Parameter (Not shown in equation)



Significance Level (α)	Confidence Level (100 - α)	Two-tailed Sign. Level ($\alpha/2$)	$Z_{\alpha/2}$
0.01	99%	0.005	2.575
0.05	95%	0.025	1.960
0.10	90%	0.05	1.645

Significance Level (α)	Confidence Level (100 - α)	Z_{α}
0.01	99%	2.33
0.05	95%	1.65
0.10	90%	1.28

Discrete Data Distribution

Distribution	Mean	Standard Deviation	Probability	Parameters
The Binomial Distribution	$\mu = n * p$	$\sigma = \sqrt{n * p(1 - p)}$	$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$	n is the sample size p is the probability of success $\binom{n}{x} = \frac{n!}{x!(n-x)!}$
The Poisson Distribution	$\mu = \lambda$	$\sigma = \sqrt{\lambda}$	$P(X = x) = \frac{e^{-\lambda} * \lambda^x}{x!}$	e = Euler's Number = 2.71828... λ = mean number of occurrences
The Hypergeometric Distributions	NA	NA	$f(x) = \frac{\binom{A}{x} * \binom{N-A}{n-x}}{\binom{N}{n}}$	N is the population quantity n is the sample quantity A is the number of "Successes" in the population $\binom{n}{x} = \frac{n!}{x!(n-x)!}$

Sampling Distribution

Distribution	Test Statistic	Parameters	Usage
The F Distribution	$F = \frac{(S_1)^2}{(S_2)^2}$	(S₁)² is the first sample variance (S₂)² is the second sample variance	Hypothesis Testing, ANOVA Analysis
Student T Distribution	$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$	\bar{x} is the sample mean, μ is the population mean s is the sample standard deviation n is the sample size	Confidence Intervals, Hypothesis Testing
The Chi-Squared Distribution	$X^2 = \frac{(N-1)s^2}{\sigma^2}$ or $X^2 = \sum_i^{i=k} \frac{(O_i - E_i)^2}{E_i}$	N is the sample size, s² is the sample variance σ² is the population variance O_i = Observed Value, E_i = Expected Value	Confidence Intervals, Hypothesis Testing, Contingency Tables, Goodness of Fit Testing

Critical Values of the Student T's Distribution at Various Degrees of Freedom and Alpha Levels (α)

df (v)	α = 0.1	α = 0.05	α = 0.025	α = 0.01	α = 0.005	α = 0.001
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144

df (v)	α = 0.1	α = 0.05	α = 0.025	α = 0.01	α = 0.005	α = 0.001
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552

Confidence Intervals

Parameter of Interest	Equation	Parameters	Use When
The Population Mean (μ)	$\bar{x} \pm Z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}$	\bar{x} is the sample mean, σ is the population standard deviation, n is the sample size, $Z_{\frac{\alpha}{2}}$ is the z-statistic associated with the confidence level	The population variance is known and the sample size (n) is greater than 30
The Population Mean (μ)	$\bar{x} \pm t_{\frac{\alpha}{2}} * \frac{s}{\sqrt{n}}$	\bar{x} is the sample mean, s is the sample standard deviation, n is the sample size, $t_{\frac{\alpha}{2}}$ is the t-statistic associated with the confidence level	The population variance is unknown or the sample size (n) is less than 30
The Population Variance	$\frac{(n-1)s^2}{X_{1-\alpha/2}^2} < \sigma^2 < \frac{(n-1)s^2}{X_{\alpha/2}^2}$	n is the sample size, s^2 is the sample variance, $X_{1-\alpha/2}^2$ and $X_{\alpha/2}^2$ are the critical chi-squared values associated with the confidence level	Creating a confidence interval for the population variance when the sample variance is known.
The Population Standard Deviation	$\sqrt{\frac{(n-1)s^2}{X_{1-\alpha/2}^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{X_{\alpha/2}^2}}$	n is the sample size, s^2 is the sample variance, $X_{1-\alpha/2}^2$ and $X_{\alpha/2}^2$ are the critical chi-squared values associated with the confidence level	The square root of variance.
The Population Proportion	$p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p * (1-p)}{n}}$	p is the sample proportion, n is the sample size, and $Z_{\frac{\alpha}{2}}$ is the z-statistic associated with the confidence level	Creating a confidence interval for the population proportion when the sample proportion is known.

Hypothesis Testing

Parameter of Interest	Equation	Parameters	Use When
Population Mean	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$	\bar{x} is the sample mean, μ is the population mean, σ is the population standard deviation, n is the sample size	The population variance is known and the sample size (n) is greater than 30
Population Mean	$t - statistic = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$	\bar{x} is the sample mean, μ is the population mean, s is the sample standard deviation; n is the sample size	The population variance is unknown or the sample size (n) is less than 30
Population Variance	$X^2 = \frac{(n-1)s^2}{\sigma^2}$	n is the sample size, S^2 is the sample variance, σ^2 is the population variance	Comparing a sample variance against a population variance
Population Variance	$F = \frac{s_1^2}{s_2^2}$	$(S_1)^2$ is the first sample variance $(S_2)^2$ is the second sample variance	Comparing two population variances against each other
Population Proportions	$Z_o = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}}$	p_o = the hypothesized population proportion, \hat{p} is the sample proportion, n is the sample size,	Comparing a sample proportion against a population proportion

Goodness of Fit Testing

Parameter of Interest	Equation	Parameters	Use When
Chi-Squared	$X^2 = \sum_i^{i=k} \frac{(O_i - E_i)^2}{E_i}$	O_i = Observed Value from sample data, E_i = Expected Value from the assumed population.	Comparing sample data (observed values) against the expected values from an assumed population.

Contingency Tables

Parameter of Interest	Equation	Parameters	Use When
Chi-Squared	$X^2 = \sum_i^{i=k} \frac{(O_i - E_i)^2}{E_i}$	O_i = Observed Value from the sample data $E_i = \frac{Row_{total} * Column_{total}}{Grand Total} = \frac{R * C}{N}$	Comparing observed values from a contingency table against the expected values (E_i) to determine if the two factors are independent. $df_{contingency} = (r - 1)(c - 1)$

ANOVA

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	Sum of Squares of the Treatment (SS_t)	DF of the Treatment (DF_t)	Mean Squares of the Treatment (MST)	= MST / MSE
Error (Within)	Sum of Squares of the Error (SS_e)	DF of the Error (DF_e)	Mean Squares of the Error (MSE)	
Total	Total Sum of Squares (SS_{total})	Total DF (DF_{Total})		

$SS_{total} = \sum (X_i - GM)^2 = SS_e + SS_t$ $SS_e = \sum (X_i - \bar{X})^2$ $SS_t = \sum n(\bar{X}_i - GM)^2 \quad \text{where GM = Grand Mean}$
$\text{Mean Square of the Treatment} = \frac{SS_{Treatment}}{DF_{Treatment}}$ $\text{Mean Square of the Error} = \frac{SS_{Error}}{DF_{Error}}$ $F - \text{statistic} = \frac{MST}{MSE}$

$DF_{total} = DF_{error} + DF_{treatment} = N - 1$	$N = \text{total observations}$
$DF_{treatment} = a - 1$	$a = \text{Number of treatments}$
$DF_{error} = (N-1) - (a-1) = a(n-1)$	$n = \text{samples per treatment group}$

Relationships Between Variables

$$\hat{y} = \beta_1 x + \beta_0 + \varepsilon$$

$$\text{Slope} = \beta_1 = \frac{S_{xy}}{S_{x^2}}$$

$$Y - \text{Intercept} = \beta_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\varepsilon = y - \hat{y}$$

$$S_{xy} = \sum(X_i * Y_i) - \frac{(\sum X_i)(\sum Y_i)}{n}$$

$$\bar{X} = \frac{\sum X_i}{n}$$

$$S_{y^2} = \sum Y_i^2 - \frac{(\sum Y_i)^2}{n}$$

$$SSE = \sum (y - \hat{y})^2 \quad \text{Or} \quad SSE = S_{yy} - \beta_1 S_{xy}$$

$$\text{Sample Variance of the Error: } s_e^2 = \frac{SSE}{n-2}$$

$$S_{x^2} = \sum X_i^2 - \frac{(\sum X_i)^2}{n}$$

$$\bar{Y} = \frac{\sum Y_i}{n}$$

$$S_{x^2} = \sum X_i^2 - \frac{(\sum X_i)^2}{n}$$

Hypothesis Testing for Linear Relationship

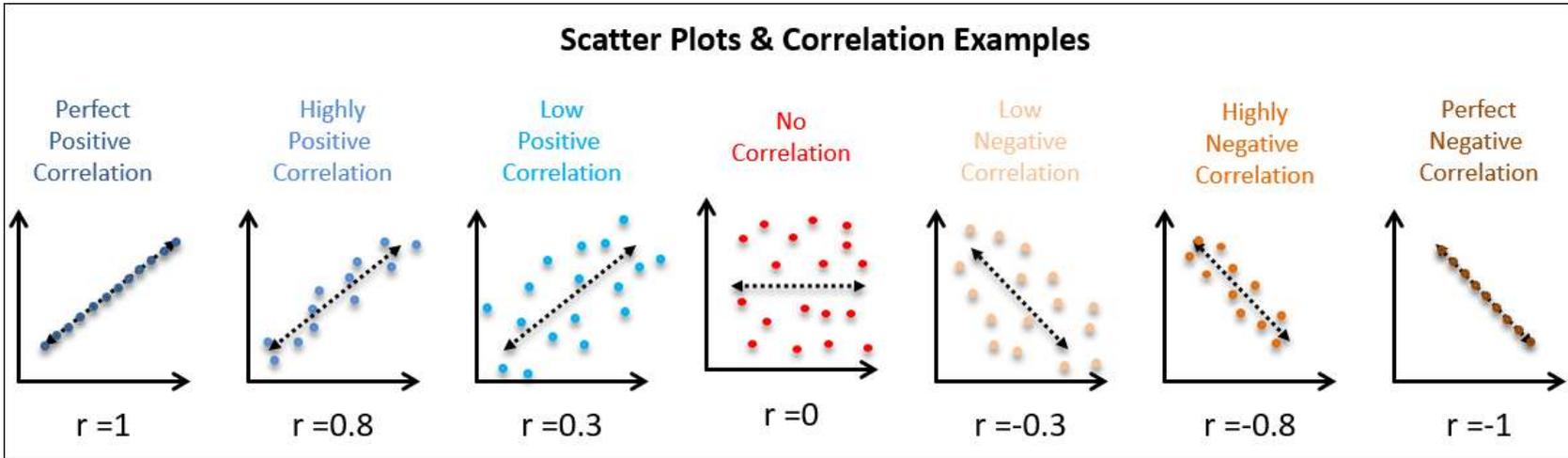
$$H_0: \beta_1 = 0 \quad \& \quad H_A: \beta_1 \neq 0$$

$$t = \frac{\hat{\beta}_1 - \beta_1}{S_e / \sqrt{S_{xx}}}$$

degrees of Freedom d.f. = n - 2

Correlation Coefficients

Parameter	Equation	Description
r	$r_{xy} = \frac{S_{xy}}{\sqrt{S_{xx}} * \sqrt{S_{yy}}}$	The Pearson Correlation Coefficient, r , measures the strength of the linear relationship between two variables which means that a change in one variable (the independent variable) also implies a change in the other (dependent variable).
R ²	$R^2 = r_{xy}^2$	The coefficient of determination, R² , reflects the proportion of the total variability in the Y variable that can be explained by the regression line, and thus it also reflects the adequacy of your model for a particular sample of data.



Statistical Process Control

The X-BAR AND R CHART

$$\text{The Grand Mean} = \bar{\bar{X}} = \frac{\sum \bar{X}_i}{k} = \frac{\text{Sum of Subgroup Average Values}}{\# \text{ of Subgroups}}$$

$$\text{X - bar Control Limits: } UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} \quad LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

$$\text{The Average Range} = \bar{R} = \frac{\sum R_i}{k} = \frac{\text{Sum of Subgroup Ranges}}{\# \text{ of Subgroups}}$$

$$\text{The R Chart Control Limits: } UCL_R = D_4\bar{R} \quad LCL_R = D_3\bar{R}$$

$$\text{Population Standard Deviation} = \hat{\sigma} = \frac{\bar{R}}{d_2}$$

The X-BAR AND S CHART

$$\text{The Grand Mean: } \bar{\bar{X}} = \frac{\sum \bar{X}_i}{k} = \frac{\text{Sum of Subgroup Average Values}}{\# \text{ of Subgroups}}$$

$$\text{X - bar Chart Control Limits: } UCL_{\bar{X}} = \bar{\bar{X}} + A_3\bar{s} \quad LCL_{\bar{X}} = \bar{\bar{X}} - A_3\bar{s}$$

$$\text{The Average Sample Standard Deviation: } \bar{s} = \frac{\sum s_i}{k} = \frac{\text{Sum of Subgroup St. Dev.}}{\# \text{ of Subgroups}}$$

$$\text{The S Chart Control Limits: } UCL_s = B_4\bar{s} \quad LCL_s = B_3\bar{s}$$

$$\text{Population Standard Deviation} = \hat{\sigma} = \frac{\bar{s}}{C_4}$$

X-Bar and R Chart

Subgroup Sample Size	X-Bar Factor	Range Factors		Variance Factor
		D ₃	D ₄	
n	A ₂	D ₃	D ₄	d ₂
2	1.880	-	3.267	1.128
3	1.023	-	2.575	1.693
4	0.729	-	2.282	2.059
5	0.577	-	2.115	2.326
6	0.483	-	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
15	0.223	0.347	1.653	3.472

X-Bar and S Chart

Subgroup Sample Size	X-Bar Factor	Standard Deviation Factors		Variance Factor
		B ₃	B ₄	
n	A ₃	B ₃	B ₄	c ₄
2	2.659	-	3.267	0.7979
3	1.954	-	2.568	0.8862
4	1.628	-	2.266	0.9213
5	1.427	-	2.089	0.9400
6	1.287	0.030	1.970	0.9515
7	1.182	0.118	1.882	0.9594
8	1.099	0.185	1.815	0.9650
9	1.032	0.239	1.761	0.9693
10	0.975	0.284	1.716	0.9727
15	0.789	0.428	1.572	0.9823
20	0.680	0.510	1.490	0.9869
25	0.606	0.565	1.435	0.9896

The I-MR Chart

$$\text{Individual Chart Centerline} = \bar{X} = \frac{\sum X_i}{k} = \frac{\text{Sum of Individual Values}}{\# \text{ of Individual Values}}$$

$$\text{Individual Chart Control Limits: } UCL_I = \bar{X} + E_2 \overline{MR} \quad LCL_I = \bar{X} - E_2 \overline{MR}$$

$$\text{Moving Range } (\overline{MR}) \text{ Centerline} = \frac{\sum MR_i}{k - 1} = \frac{\text{Sum of Moving Ranges}}{\# \text{ of MR's}}$$

$$\text{Moving Range Control Limits: } UCL_{MR} = D_4 \overline{MR} \quad LCL_{MR} = D_3 \overline{MR}$$

I-MR Chart				
Subgroup Sample Size	Individual Factor	Moving Range Factors		Variance Factor
n	E ₂	D ₃	D ₄	d ₂
2	2.660	-	3.267	1.128
3	1.772	-	2.575	1.693
4	1.457	-	2.282	2.059
5	1.290	-	2.115	2.326
6	1.184	-	2.004	2.534
7	1.109	0.076	1.924	2.704
8	1.054	0.136	1.864	2.847
9	1.010	0.184	1.816	2.970
10	0.975	0.223	1.777	3.078

Attribute Data Control Charts

np & p charts trend defectives and are based on the **Binomial distribution** which operates under the assumption that every unit inspected can only be counted as "bad" one time.

p-Char trends *defectives* with a *Variable* Sample Size

np-Chart trends *defectives* with a *Constant* Sample Size

u & c Charts utilize the **Poisson distribution** as they trend the number of defects where it is possible for each item inspected to contain multiple defects.

u-Chart trends *defects* with a *Variable* Sample Size

c-Chart trends *defects* with a *Constant* Sample Size

		Sample Size	
		Constant	Variable
Type	Defect	c Chart	u Chart
	Defectives	np Chart	p Chart

Attribute Data Control Charts

The p Chart

$$\bar{p} = \text{Centerline} = \frac{\sum np}{\sum n} = \frac{\text{Sum of All Defectives}}{\text{Sum of Subgroup Quantity}}$$

$$UCL_{\bar{p}} = \bar{p} + 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} \quad LCL_{\bar{p}} = \bar{p} - 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$

$$\bar{n} = \text{Average Sample Size} = \frac{\sum n}{k} = \frac{\text{Sum of subgroup quantity}}{\# \text{ of subgroups}}$$

The np Chart

$$n\bar{p} \text{ Centerline} = \frac{\sum np}{k} = \frac{\text{Sum of All Defectives}}{\# \text{ of subgroups}}$$

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \quad LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\bar{p} = \% \text{ Defective} = \frac{\sum np}{\sum n} = \frac{\text{Sum of All Defectives}}{\text{Sum of Subgroup Quantity}}$$

The u Chart

$$\bar{u} = \text{Centerline} = \frac{\sum c}{\sum n} = \frac{\text{Sum of All Defects}}{\text{Sum of units inspected}}$$

$$\bar{n} = \text{Average of Samples per Subgroup} = \frac{\sum n}{k} = \frac{\text{Sum of units inspected}}{\text{Number of subgroups}}$$

$$UCL_u = \bar{u} + 3 \sqrt{\frac{\bar{u}}{\bar{n}}} \quad LCL_u = \bar{u} - 3 \sqrt{\frac{\bar{u}}{\bar{n}}}$$

The c Chart

$$\bar{c} = \text{Centerline} = \frac{\sum c}{k} = \frac{\text{Sum of All Defects}}{\# \text{ of Subgroups}}$$

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} \quad LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

		Sample Size	
		Constant	Variable
Type	Defect	c Chart	u Chart
	Defectives	np Chart	p Chart

Process Capability

$$C_p = \frac{USL - LSL}{6s_{c_p}} \quad C_r = \frac{1}{C_p} \quad C_{pk} = \text{Min}(C_{p,Lower}, C_{p,Upper}) = \text{Min}\left(\frac{USL - \bar{x}}{3s_{c_p}}, \frac{\bar{x} - LSL}{3s_{c_p}}\right)$$

$$s_{c_p} = \frac{R}{d_2} \text{ (if using an } \bar{X} - R \text{ Chart)} \quad \text{or} \quad s_{c_p} = \frac{s}{c_4} \text{ (if using an } \bar{X} - S \text{ Chart)}$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{s^2 + (\bar{x} - T)^2}}$$

Process Performance Indices

$$P_p = \frac{USL - LSL}{6s_{p_p}} \quad P_{pk} = \text{Min}\left(\frac{USL - \bar{x}}{3s_{p_p}}, \frac{\bar{x} - LSL}{3s_{p_p}}\right) \quad s_{p_p} = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

Design of Experiments

Treatments in a DOE

Full Factorial Design: Number of Treatments = Levels^{Factors} = $L^F = 2^F$

Half Factorial Design: Number of Treatments = $\frac{\text{Levels}^{\text{Factors}}}{2} = \frac{L^F}{2} = \frac{2^F}{2} = 2^{F-1}$

Quarter Factorial Design: Number of Treatments = $\frac{\text{Levels}^{\text{Factors}}}{4} = \frac{L^F}{4} = \frac{2^F}{2^2} = 2^{F-2}$

# of Factors	Full Factorial Exp.	Half Factorial Exp.	Quarter Factorial Exp.
2	4	2	1
3	8	4	2
4	16	8	4
5	32	16	8
6	64	32	16
7	128	64	32
8	256	128	64
9	512	256	128
10	1024	512	256

Calculating Effects from a DOE

Estimated Effect = Average at High – Average at Low

Reliability Equations

The Exponential Distribution

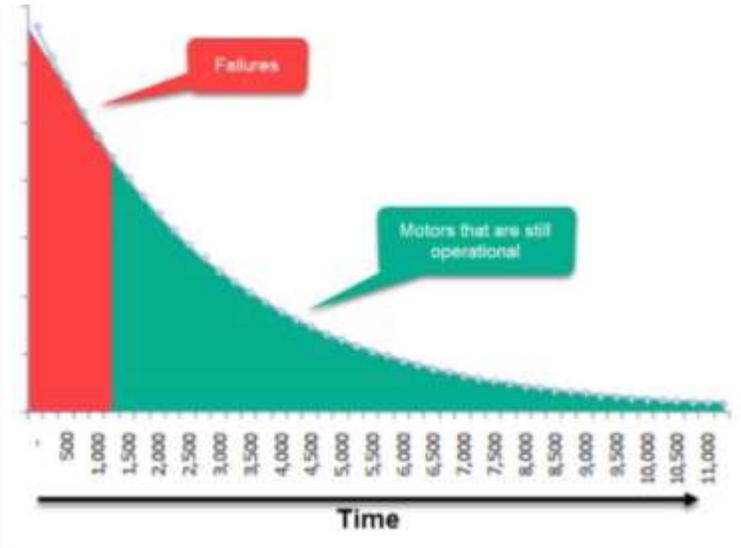
Reliability: $R(t) = e^{-\lambda t} = e^{-\frac{t}{\theta}}$ **Failures:** $F(t) = 1 - e^{-\lambda t}$ **PDF:** $f(t) = \lambda e^{-\lambda t}$

Failure Rate $= \lambda = \frac{1}{\theta} = \frac{1}{MTTF} = \frac{\text{Number of Failures}}{\text{Operating Time (Cycles)}}$

$\theta = \text{MTBF (or MTTF)} = \frac{1}{\lambda} = \frac{\text{Operating Time (Cycles)}}{\text{Number of Failures}}$

Mean Time to Failure (**MTTF**) is only used when an item is not repairable.

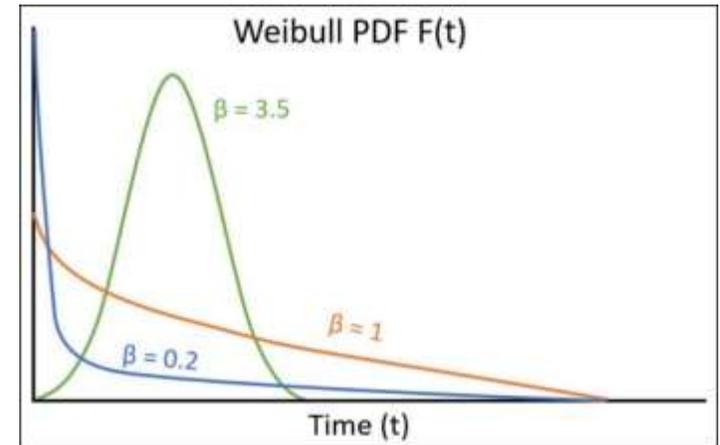
Mean Time Between Failures (**MTBF**) is used if a product or asset is repairable.



Weibull Distribution Equations

PDF: $f(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^\beta}$ **Reliability:** $R(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}$

CDF: $F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^\beta}$ **Failure Rate:** $h(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1}$



Weibull Parameters

- β(Beta)** - the Weibull Shape or Slope Parameter
- θ(Theta)** - the Weibull Scale Parameter
- Δ(Delta)** - The Weibull Location Parameter (Not shown above)

Maintainability and Availability

$MTTR = \frac{\text{Total Time Spent in Maintenance}}{\text{Total Number of Maintenance Activities conducted}} = \text{Maintainability}$

$\text{Availability} = \frac{MTBF}{MTBF + MTTR}$

Reliability and Fault Tree Equations

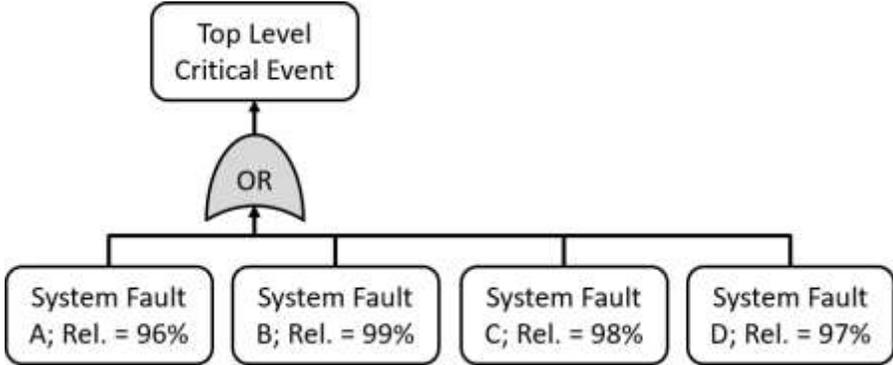
Series System Reliability

Series System Reliability = $R_{system} = R_1 \times R_2 \times R_3 \times R_4 \times \dots \times R_n$



Fault Tree Reliability (OR GATE)

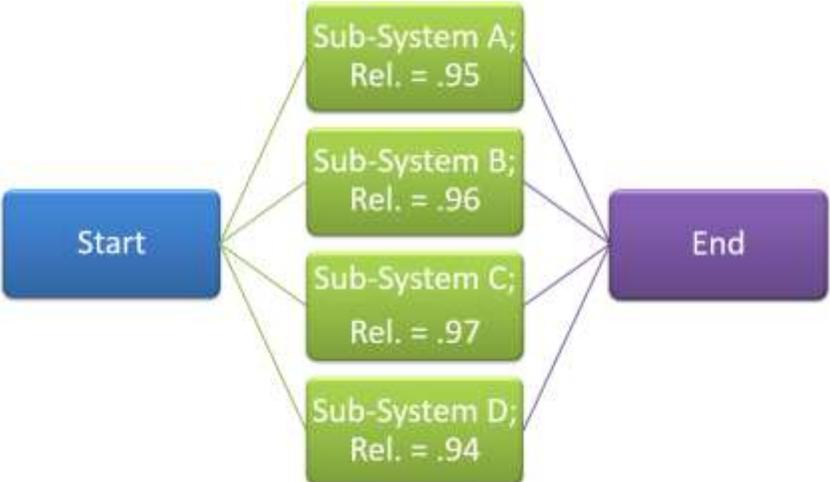
Reliability of OR Gate: $R_{OR\ Gate} = R_1 \times R_2 \times R_3 \times R_4 \times \dots \times R_n$



Parallel System Reliability

Parallel System Reliability = $1 - (U_1 \times U_2 \times U_3 \times U_4 \times \dots \times U_n)$

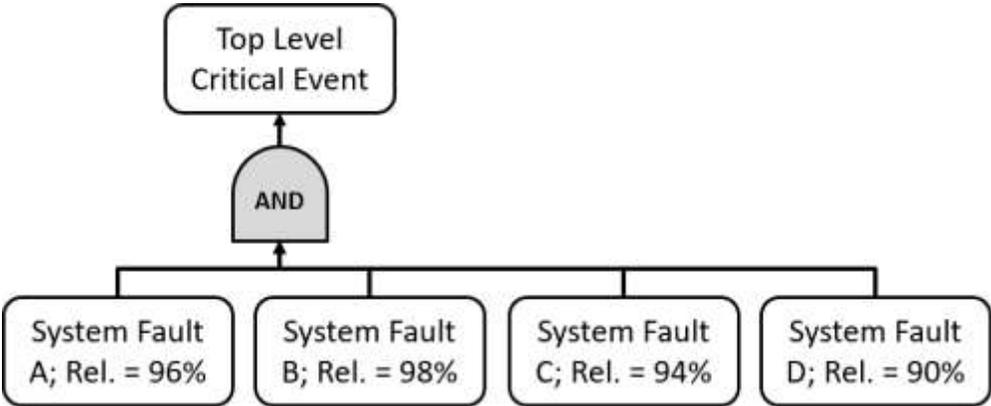
Where $U_1 = 1 - R_1$



Fault Tree Reliability (AND GATE)

AND GATE System Reliability = $1 - (U_1 \times U_2 \times U_3 \times U_4 \times \dots \times U_n)$

Where $U_1 = 1 - R_1$



Gauge R&R (Measurement System Analysis)

The Average and Range Method

$$\text{Equipment Variation (EV)} = \sigma_{\text{Repeatability}} = \frac{\bar{R}}{d_2}$$

$$\text{Appraiser Variation (AV)} = \sigma_{\text{Reproducibility}} = \sqrt{\left(\frac{R_o}{d_2}\right)^2 - \frac{(EV)^2}{n * r}}$$

$$\text{Part to Part Variation (PV)} = \sigma_p = \frac{R_p}{d_2}$$

$$\text{Measurement System Variation (GR\&R)} = \sigma_{GRR} = \sqrt{(EV)^2 + (AV)^2}$$

$$\text{Total Variation} = \sqrt{(GR\&R)^2 + (PV)^2}$$

The Precision/Tolerance (P/T) Ratio

$$\frac{P}{T} \text{ Ratio} = \frac{\text{Total Measurement System Variation}}{\text{Tolerance}}$$

$$\frac{P}{T} \text{ Ratio} = \frac{6 * \sigma_{GRR}}{\text{Upper Spec.} - \text{Lower Spec.}}$$

The Range Method

$$\text{Measurement System Variation (GRR)} = \sigma_{GRR} = \frac{\bar{R}}{d_2^*}$$

The ANOVA Method

$$\text{Repeatability} - \text{Equipment Variation (EV)}: \sigma_{\text{Repeatability}}^2 = MS_E$$

$$\text{Repeatability} - \text{Appraiser Variation (AV)}: \sigma_{\text{Reproducibility}}^2 = \frac{MS_O - MS_{PxO}}{P * R}$$

$$\text{Appraiser} - \text{Part Interaction Variation (I}_{PxO}): \sigma_{\text{Interaction}}^2 = \frac{MS_{PxO} - MS_E}{R}$$

$$\text{Part Variation (PV)}: \sigma_{\text{Part}}^2 = \frac{MS_P - MS_{PxO}}{O * R}$$

$$\text{Measurement System Variation (GRR)} = \sigma_{GRR} = \sqrt{(EV)^2 + (AV)^2 + (I_{PxO})^2}$$

$$\text{Total Variation} = \sqrt{(GRR)^2 + (PV)^2}$$

The Percent Variation Calculation

$$\text{Percent Variation} = \frac{\text{Measurement System Variation}}{\text{Total Variation}}$$

$$\sigma_{\text{Total Variation}}^2 = \sigma_{GR\&R}^2 + \sigma_{\text{Part To Part Variation}}^2$$

$$\text{Equipment Variation (EV - Repeatability)} = \frac{EV}{TV} * 100 = \frac{\text{Equipment Variation (Repeatability)}}{\text{Total Variation}}$$

$$\text{Appraiser Variation (AV - Reproducibility)} = \frac{AV}{TV} * 100 = \frac{\text{Appraiser Variation (Reproducibility)}}{\text{Total Variation}}$$

$$\text{Measurement System Variation} = \frac{GRR}{TV} * 100 = \frac{\text{Gauge R\&R (Repeatability + Reproducibility)}}{\text{Total Variation}}$$

$$\% \text{ Part to Part Variation} = \frac{PV}{TV} * 100 = \frac{\text{Part to Part Variation (PV)}}{\text{Total Variation}}$$

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This free, 10-day course covers the top 10 topics in the CQE Exam and includes daily video lectures and practice quizzes for the 10 most important topics on the CQE Exam!!



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All About Confidence Intervals

Confidence Interval - Population Mean

Interval Estimate of Population Mean (known variance) : $\bar{x} \pm Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

- n = sample size
- \bar{x} = sample mean
- σ = population standard deviation
- $Z_{\frac{\alpha}{2}}$ = Z-score associated with alpha risk

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-Andy

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