

## Question #1: If the probability of a success on a single trial is 0.2 and 3 trials are performed, what is the probability of at least one success?

Here's how I would answer that problem.

First is, I think about the variables. We're given the probability of success of a given outcome, and we're given a fixed number of independent trials.

These 2 variables, and the problem statement are testing our knowledge of the various probability distributions, and the right one given these variables is the binomial distribution.

Then, the way I think about it is, there are 3 trials, and those trial can only have 4 outcomes:

- 3 out of the 3 trials are successful ( $X = 3$ )
- 2 out of the 3 are successful ( $X = 2$ )
- 1 out of the 3 is successful ( $X = 1$ )
- 0 out of the 3 are successful ( $X = 0$ )

So to calculate the probability of *having at least one trial be successful*, you can calculate the cumulative probability of  **$X = 3$  and  $X = 2$  and  $X = 1$**

OR you can calculate the probability that  $X = 0$ , and then take the inverse (or complement), because we know that the probability of all 4 of those outcomes have to equal up to 100%, or 1.0.

So, I've used the binomial probability equation to show you how to calculate the probability of all 4 outcomes, so you can see what I'm talking about.

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x} \quad \text{where} \quad \binom{n}{x} = \frac{n!}{x! (n - x)!}$$

In terms of the variables,  $n = 3$ , because there are 3 independent trials, and  $p = 0.2$  because that's the probability of success.

$$P(X = 3) = \binom{3}{3} 0.2^3 (1 - 0.2)^{3-3} = 1 * 0.008 * 1 = 0.008$$

$$P(X = 2) = \binom{3}{2} 0.2^2 (1 - 0.2)^{3-2} = 3 * 0.04 * 0.8 = 0.096$$

$$P(X = 1) = \binom{3}{1} 0.2^1 (1 - 0.2)^{3-1} = 3 * 0.2 * 0.64 = 0.384$$

Cumulatively, these 3 events add up to:

$$P(X = 3) + P(X = 2) + P(X = 1) = 0.008 + 0.096 + 0.384 = 0.488 \text{ or } 48.8\% \text{ Probability}$$

Now, the easier way to calculate this, is to calculate the probability that ZERO of the trials were successful, and then just take the complement of that.

$$P(X = 0) = \binom{3}{0} 0.2^0 (1 - 0.2)^{3-0} = 1 * 1 * 0.512 = 0.512$$

Then we just take the complement (or inverse)

$$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.512 = 0.488 \text{ or } 48.8\%$$