

Problem Set For ANOVA:

1. Identify all of the statements below regarding ANOVA that are false:

- in ANOVA, you're most likely to accept the null hypothesis when MST/MSE is high
- Whether the null hypothesis is true or not, the MSE (Mean Square of the Error) is a good approximation of the population variance.
- In ANOVA Analysis, the response variable that you're measuring is known as the independent variable.
- If you were to run an experiment studying muffins when you vary the baking temperature to 400, 425 and 450. This experiment could be described as having 3 levels of one factor.

2. Identify all of the statements below regarding ANOVA that are true:

- If you were to run an experiment that studied the effect on muffin moisture, based on temperature and baking duration, the ANOVA analysis would be a one way ANOVA
- For an ANOVA analysis with 5 treatment groups and 3 measurements per group the degrees of freedom of the error is 10.
- treatment variation is a reflection of the variation within each treatment group.
- Homogeneity of variance is a baseline assumption required for ANOVA analysis to be accurate.

3. Identify all of the statements below regarding ANOVA that are false:

- The Treatment Variation is the variability within the data set that can be attributed to the difference between the different treatment groups.
- The error variation in ANOVA grows if the sample groups have mean values that diverge from each other.
- The dependent variable in ANOVA is called a Factor.
- If you were to run an experiment that studied the effect on muffin moisture based on baking temperature, the independent variable would be the oven temperature.
- In ANOVA Analysis, if the null hypothesis is false then the F-statistic will be equal or less than 1.

4. Identify all of the statements below regarding ANOVA that are true:

- The null hypothesis of ANOVA Analysis is situational and depends on the data being studied
- Homogeneity of variance is a baseline assumption required for ANOVA analysis to be accurate.
- The normality assumption associated with a One Way ANOVA is not required for a Two Way ANOVA.
- ANOVA is a type of Hypothesis Test used to test hypothesis about Mean values. (True)
- If the null hypothesis is true then the Error Mean Square is an approximate estimator of the population variance.

5. Identify all of the statements below regarding ANOVA that are false:

- In ANOVA Analysis, if the null hypothesis is false then the treatment variation can be fully explained by the random nature of the data.
- The alternative hypothesis of ANOVA analysis is that the means are all different
- In ANOVA Analysis, the sum of squares is calculated by dividing the mean square by the degrees of freedom.
- With a Two Way ANOVA, interactions can be measured but only if there is more than one replicate per treatment group.
- In ANOVA analysis, the total sum of squares can be calculated by adding up the sum of squares of the error and the treatment.

6. Identify all of the statements below regarding ANOVA that are True:

- If the MSE and MSB are approximately the same, it is highly likely that null hypothesis will be rejected.
- For an ANOVA analysis with 4 treatment groups and 5 measurements per group, the degrees of freedom of the treatment is 19.
- The Error Variation within ANOVA is the variability that can be attributed to the random error associated with the response variable.
- If the null hypothesis is false then the Treatment Mean Square is an approximate estimator of the population variance.

7. Identify all of the statements below regarding ANOVA that are True:

- Any difference among the population means in the analysis of variance will inflate the error sum of squares
- The t-statistic in ANOVA is calculated by dividing the error mean square by the treatment mean square
- If the null hypothesis is true then MSE & MST will be approximate equal and the F-statistic will equal ~ 1
- A Two Way ANOVA occurs when 1 factor is varied at 2 levels

8. Which distribution is used to make the accept/reject decision for ANOVA Analysis:

- chi-squared distribution
- T-distribution
- Binomial distribution
- Normal distribution
- None of the above

9. Which statistical tool should be used to test the equality of 3 or more population means?

- ANOVA
- Interval Estimate
- Chi-Squared Test
- T-Test
- None of the above

10. You're performing an ANOVA Analysis, and the total sum of squares is 36 and the treatment sum of squares is 16, what would the error sum of squares be?

- 20
- 52
- 30
- 16
- Not Enough Information

11. You're performing an ANOVA Analysis, and the treatment sum of squares is 12 and the error sum of squares is 18, what is the total sum of squares?

- 6
- 36
- 30
- 18
- Not Enough Information

12. You're performing an ANOVA Analysis, and the total sum of squares is 24 and the error sum of squares is 16, what would the treatment sum of squares be?

- 40
- 23
- 15
- 8
- Not Enough Information

13. Match the following equations below is their correct "Sum of Squares" Description (Treatment, Error & Total sum of Squares):

- $\sum(X_i - \bar{X})^2$
- $\sum(X_i - GM)^2$
- $\sum n(\bar{X}_1 - GM)^2$

14. You've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	12	13	13
Sample #2	14	15	16
Sample #3	16	17	19

Calculate the sum of squares of the Error:

- 40
- 14
- 34
- 6

15. You've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	84	87	90
Sample #2	85	88	91
Sample #3	86	89	92

Calculate the sum of squares of the Error:

- 66
- 60
- 56
- 54
- 16
- 14
- 6

16. You've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	12	13	13
Sample #2	14	15	16
Sample #3	16	17	19

Calculate the sum of squares of the Treatment:

- 40
- 14
- 34
- 6
- 2

17. You've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	84	87	90
Sample #2	85	88	91
Sample #3	86	89	92

Calculate the sum of squares of the Treatment:

- 66
- 60
- 56
- 54
- 16
- 14
- 6

18. you've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	12	13	13
Sample #2	14	15	16
Sample #3	16	17	19

Calculate the total sum of square:

- 16
- 34
- 40
- 6

19. you've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	84	87	90
Sample #2	85	88	91
Sample #3	86	89	92

Calculate the total sum of square:

- 66
- 60
- 56
- 54
- 16
- 14
- 6

20. In ANOVA Analysis, the degrees of freedom associated with the numerator of the F-value can be calculated using which equation:

- $N-1$
- $n(a)-1$
- $n(a-1)$
- $(N-1) - (a-1)$
- $a - 1$

21. You're performing an ANOVA Analysis of 1 independent variable at 4 levels and you're measuring 5 samples per level. What is the total degrees of freedom?

- 20
- 16
- 19
- 4
- 3

22. You're performing an ANOVA Analysis of 1 independent variable at 4 levels where the total degrees of freedom is 24, what is the error degrees of freedom?

- 23
- 21
- 20
- 3
- Not Enough Information

23. You're performing an ANOVA Analysis of 1 independent variable at 6 levels where the total degrees of freedom is 24, what is the treatment degrees of freedom?

- 23
- 5
- 20
- 3
- Not Enough Information

24. Match the following ANOVA Analysis degrees of Freedom (Total, Treatment & Error) to their correct equation:

$$= a*n - 1$$

$$= a - 1$$

$$= a(n-1)$$

25. You're performing an ANOVA Analysis of 1 independent variable at 3 levels and you're measuring 10 samples per level. What is the total degrees of freedom?

- 2
- 27
- 9
- 30
- 29
- Not Enough Information

26. You're performing an ANOVA Analysis of 1 independent variable at 7 levels where the total degrees of freedom is 35, what is the error degrees of freedom?

- 34
- 28
- 29
- 6
- 35
- 42
- Not Enough Information

27. You're performing an ANOVA Analysis of 1 independent variable at 10 levels where the total degrees of freedom is 20, what is the treatment degrees of freedom?

- 20
- 10
- 9
- 30
- 11
- Not Enough Information

28. The one way ANOVA Analysis below has 10 treatment groups with the total degrees of freedom of 19.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)				
Error (Within)	55			
Total	100	19		

Calculate the Treatment Mean Square for this ANOVA Table.

- 4.5
- 5
- 5.5
- 6.1

29. The one way ANOVA Analysis below has 15 treatment groups with the total degrees of freedom of 45.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	30			
Error (Within)				
Total	45	45		

Calculate the Error Mean Square for this ANOVA Table.

- 1.76
- 2.14
- 1.07
- 0.88
- 2.43

30. The one way ANOVA Analysis below has 3 treatment groups with the total degrees of freedom of 50.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	20			
Error (Within)				
Total	100	50		

What is the decision for this ANOVA Analysis if we choose the alpha risk to be 10%?

- Fail to Reject the Null
- Reject the Null
- Not Enough Information

31. The one way ANOVA Analysis below has 7 treatment groups with the total degrees of freedom of 38.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)				
Error (Within)	66			
Total	84	38		

What is the decision for this ANOVA Analysis if we use choose alpha risk to be 10%?

- Fail to Reject the Null
- Reject the Null
- Not Enough Information

Solutions for ANOVA:

1. Identify all of the statements below regarding ANOVA that are false:

- in ANOVA, you're most likely to ~~accept~~ **reject** the null hypothesis when MST/MSE is high (False)
- Whether the null hypothesis is true or not, the MSE (Mean Square of the Error) is a good approximation of the population variance. (True)
- In ANOVA Analysis, the response variable that you're measuring is known as the ~~independent~~ **dependent** variable. (False)
- If you were to run an experiment studying muffins when you vary the baking temperate to 400, 425 and 450. This experiment could be described as having 3 levels of one factor. (True).

2. Identify all of the statements below regarding ANOVA that are true:

- If you were to run an experiment that studied the affect on muffin moisture, based on temperature and baking duration, the anova analysis would be a ~~one two~~ **two way** anova (False).
- For an ANOVA analysis with 5 treatment groups and 3 measurements per group the degrees of freedom of the error is 10. (True, $a(n-1) = 5(3-1) = 10$).
- ~~treatment error~~ variation is a reflection of the variation within each treatment group. (False)
- Homogeneity of variance is a baseline assumption required for ANOVA analysis to be accurate.(True)

3. Identify all of the statements below regarding ANOVA that are false:

- The Treatment Variation is the variability within the data set that can be attributed to the difference between the different treatment groups. (True)
- The ~~error treatment~~ variation in ANOVA grows if the sample groups have mean values that diverge from each other (False).
- The ~~dependent independent~~ variable in ANOVA is called a Factor. (False)
- If you were to run an experiment that studied the affect on muffin moisture based on baking temperature, the independent variable would be the oven temperature (True).
- In ANOVA Analysis, if the null hypothesis is false then the F-statistic will be **much larger than equal or less than** 1. (False)

4. Identify all of the statements below regarding ANOVA that are true:

- The null hypothesis of ANOVA Analysis is situational and depends on the data being studied (*False, the null hypothesis in ANOVA is always the same*).
- Homogeneity of variance is a baseline assumption required for ANOVA analysis to be accurate. (True)
- The normality assumption associated with a One Way ANOVA is **not** required for a Two Way ANOVA (*False, the assumption of normality also applies to Two Way ANOVA*)
- ANOVA is a type of Hypothesis Test used to test hypothesis about Mean values. (True)
- If the null hypothesis is true then the Error Mean Square is an approximate estimator of the population variance. (True)

5. Identify all of the statements below regarding ANOVA that are false:

- In ANOVA Analysis, if the null hypothesis is **false true** then the treatment variation can be fully explained by the random nature of the data. (False)
- The alternative hypothesis of ANOVA analysis is that **at least 1 mean is different all the means are all different** (False)
- In ANOVA Analysis, the **mean square sum-of-squares** is calculated by dividing the **mean-square sum of squares** by the degrees of freedom (False).
- With a Two Way ANOVA, interactions can be measured but only if there is more than one replicate per treatment group (True)
- In ANOVA analysis, the total sum of squares can be calculated by adding up the sum of squares of the error and the treatment. (True)

6. Identify all of the statements below regarding ANOVA that are True:

- If the MSE and MSB are approximately the same, it is highly **likely unlikely** that null hypothesis will be rejected. (FALSE)
- For an ANOVA analysis with 4 treatment groups and 5 measurements per group, the degrees of freedom of the treatment is **19 3**. (*False, DF of the treatment is the number of groups (4) minus 1*)
- The Error Variation within ANOVA is the variability that can be attributed to the random error associated with the response variable. (True)
- If the null hypothesis is false then the **Treatment Error** Mean Square is an approximate estimator of the population variance. (False)

7. Identify all of the statements below regarding ANOVA that are True:

- Any difference among the population means in the analysis of variance will inflate the **treatment error** sum of squares (False)
- The ~~t-statistic~~ **f-statistic** in ANOVA is calculated by dividing the ~~error~~ **treatment** mean square by the ~~error~~ **treatment** mean square (False)
- If the null hypothesis is true then MSE & MST will be approximate equal and the F-statistic will equal ~1. (True)
- A Two Way ANOVA occurs when ~~1~~ **2** factor is varied at ~~2~~ **multiple** levels (False)

8. Which distribution is used to make the accept/reject decision for ANOVA Analysis:

- chi-squared distribution
- T-distribution
- Binomial distribution
- Normal distribution
- None of the above - The **F-Distribution** is used within ANOVA.

9. Which statistical tool should be used to test the equality of 3 or more population means?

- ANOVA
- Interval Estimate
- Chi-Squared Test
- T-Test
- None of the above

10. You're performing an ANOVA Analysis, and the total sum of squares is 36 and the treatment sum of squares is 16, what would the error sum of squares be?

- 20
- 52
- 30
- 16
- Not Enough Information

$$SS_{\text{error}} = SS_{\text{total}} - SS_{\text{treatment}} = 36 - 16 = 20$$

11. You're performing an ANOVA Analysis, and the treatment sum of squares is 12 and the error sum of squares is 18, what is the total sum of squares?

- 6
- 36
- 30
- 18
- Not Enough Information

$$SS_{\text{total}} = SS_{\text{error}} + SS_{\text{treatment}}$$

$$SS_{\text{total}} = 12 + 18 = 30$$

12. You're performing an ANOVA Analysis, and the total sum of squares is 24 and the error sum of squares is 16, what would the treatment sum of squares be?

- 40
- 23
- 15
- 8
- Not Enough Information

$$SS_{\text{treatment}} = SS_{\text{total}} - SS_{\text{error}}$$

$$SS_{\text{treatment}} = 24 - 16 = 8$$

13. Match the following equations below is their correct description:

- Treatment Sum of Squares - $\sum n(\bar{X}_1 - GM)^2$
- Error Sum of Squares - $\sum (X_i - \bar{X})^2$
- Total Sum of Squares - $\sum (X_i - GM)^2$

14. You've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	12	13	13
Sample #2	14	15	16
Sample #3	16	17	19

Calculate the sum of squares of the Error:

- 40
- 14
- 34
- 6

First, we must calculate the sample mean for each treatment group (14, 15 & 16).

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	12	13	13
Sample #2	14	15	16
Sample #3	16	17	19
Sample Mean	14	15	16

Then we can calculate the error sum of squares using the following equation where we're going to compare each individual value (X_i) against the mean value for that treatment group.

$$SS_e = \sum (X_i - \bar{X})^2$$

$$SS_e = \sum (12 - 14)^2 + (14 - 14)^2 + (16 - 14)^2 + (13 - 15)^2 + (15 - 15)^2 + (17 - 15)^2 + (13 - 16)^2 + (16 - 16)^2 + (19 - 16)^2$$

Treatment group #1 is in green, #2 in purple, & #3 in red.

$$SS_e = \sum 4 + 0 + 4 + 4 + 0 + 4 + 9 + 0 + 9 = 34$$

15. You've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	84	87	90
Sample #2	85	88	91
Sample #3	86	89	92

Calculate the sum of squares of the Error:

- 66
- 60
- 56
- 54
- 16
- 14
- 6

First, we must calculate the sample mean for each treatment group (85, 88 & 91).

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	84	87	90
Sample #2	85	88	91
Sample #3	86	89	92
Sample Mean	85	88	91

Then we can calculate the error sum of squares using the following equation where we're going to compare each individual value (X_i) against the mean value for that treatment group.

$$SS_e = \sum (X_i - \bar{X})^2$$

$$SS_e = \sum (84 - 85)^2 + (85 - 85)^2 + (86 - 85)^2 + (87 - 88)^2 + (88 - 88)^2 + (89 - 88)^2 + (90 - 91)^2 + (91 - 91)^2 + (92 - 91)^2$$

Treatment group #1 is in green, #2 in purple, & #3 in red.

$$SS_e = \sum 1 + 0 + 1 + 1 + 0 + 1 + 1 + 0 + 1 = 6$$

16. You've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	12	13	13
Sample #2	14	15	16
Sample #3	16	17	19

Calculate the sum of squares of the Treatment:

- 40
- 14
- 34
- 6
- 2

First, we must calculate the **sample mean** for each treatment group (14, 15 & 16).

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	12	13	13
Sample #2	14	15	16
Sample #3	16	17	19
Sample Mean	14	15	16

Then we must calculate the **grand mean** can be found by taking the average of the averages (14, 15 & 16), or averaging all 9 parts.

The grand mean is 15.

We can then use the grand mean to calculate the treatment sum of squares by comparing the 3 sample mean values (14, 15 & 16) against the grand mean. Note that this equation takes into account the number of samples per subgroup, n (3).

$$SS_{treatment} = \sum n(\bar{X}_i - GM)^2$$

$$SS_t = \sum 3(14 - 15)^2 + 3(15 - 15)^2 + 3(16 - 15)^2 = \sum 3 + 0 + 3 = 6$$

17. You've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	84	87	90
Sample #2	85	88	91
Sample #3	86	89	92

Calculate the sum of squares of the Treatment:

- 66
- 60
- 56
- 54
- 16
- 14
- 6

First, we must calculate the sample mean for each treatment group (85, 88 & 91).

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	84	87	90
Sample #2	85	88	91
Sample #3	86	89	92
Sample Mean	85	88	91

Then we must calculate the **grand mean** can be found by taking the average of the averages (85, 88 & 91), or averaging all 9 parts.

The grand mean is 88.

We can then use the grand mean to calculate the treatment sum of squares by comparing the 3 sample mean values (85, 88 & 91) against the grand mean. Note that this equation takes into account the number of samples per subgroup, n (3).

$$SS_{treatment} = \sum n(\bar{X}_i - GM)^2$$

$$SS_t = \sum 3(85 - 88)^2 + 3(88 - 88)^2 + 3(91 - 88)^2 = \sum 27 + 0 + 27 = 54$$

18. you've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	12	13	13
Sample #2	14	15	16
Sample #3	16	17	19

Calculate the total sum of square:

- 16
- 34
- 40
- 6

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	12	13	13
Sample #2	14	15	16
Sample #3	16	17	19
Sample Mean	14	15	16

The grand mean can be found by taking the average of the averages (14, 15 & 16), or averaging all 9 parts; it is 15.

Now we can use that to calculate the total sum of squares:

$$SS_{total} = \sum (X_i - GM)^2$$

$$SS_{total} = \sum (12 - 15)^2 + (14 - 15)^2 + (16 - 15)^2 + (13 - 15)^2 + (15 - 15)^2 + (17 - 15)^2 + (13 - 15)^2 + (16 - 15)^2 + (19 - 15)^2$$

$$SS_{total} = \sum 9 + 1 + 1 + 4 + 0 + 4 + 4 + 1 + 16 = 40$$

19. you've run an experiment where you're studying 1 factor at 3 levels, with 3 replicates per factor and measuring the response variable. your data is below.

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	84	87	90
Sample #2	85	88	91
Sample #3	86	89	92

Calculate the total sum of square:

- 66
- 60
- 56
- 54
- 16
- 14
- 6

First, we must calculate the sample mean for each treatment group (85, 88 & 91).

	Treatment Group #1	Treatment Group #2	Treatment Group #3
Sample #1	84	87	90
Sample #2	85	88	91
Sample #3	86	89	92
Sample Mean	85	88	91

Then we must calculate the **grand mean** can be found by taking the average of the averages (85, 88 & 91), or averaging all 9 parts.

The grand mean is 88.

Now we can use that to calculate the total sum of squares:

$$SS_{total} = \sum (X_i - GM)^2$$

$$SS_{total} = \sum (84 - 88)^2 + (85 - 88)^2 + (86 - 88)^2 + (87 - 88)^2 + (88 - 88)^2 + (89 - 88)^2 + (90 - 88)^2 + (91 - 88)^2 + (92 - 88)^2$$

$$SS_{total} = \sum 16 + 9 + 4 + 1 + 0 + 1 + 4 + 9 + 16 = 60$$

20. In ANOVA Analysis, the degrees of freedom associated with the numerator of the F-value can be calculated using which equation:

- N-1
- $n(a)-1$
- $n(a-1)$
- $(N-1) - (a-1)$
- **a - 1**

Recall that in ANOVA, $F = MST / MSE$, with MST being the numerator of the equation. Also recall that the degrees of freedom associated with the Treatment Mean Square (MST) is equal to the number of treatment groups (a) - 1.

21. You're performing an ANOVA Analysis of 1 independent variable at 4 levels and you're measuring 5 samples per level. What is the total degrees of freedom?

- 20 (random distractor)
- 16 (distractor - error df: $a(n-1)$)
- **19 (N - 1; $4*5 - 1$)**
- 4 (distractor - treatment df: $a - 1, 5 - 1$)
- 3 (random distractor)

Recall that in ANOVA, the total degrees of freedom = $N - 1$, where $N = a*n - 1$, with a being the number of levels (treatment groups) and n being the number of replicates or samples per level (treatment group).

$$N = 4*5 - 1 = 19$$

22. You're performing an ANOVA Analysis of 1 independent variable at 4 levels where the total degrees of freedom is 24, what is the error degrees of freedom?

- 23 (distractor)
- **21 (correct - $24 - 3$)**
- 20 (distractor)
- 3 (distractor, treatment df: $a - 1$)
- Not Enough Information

First we must solve for the Treatment D.F., which is equal to $4-1$, or 3. Then we can subtract 3 from 24 to get the error degrees of freedom of 21.

23. You're performing an ANOVA Analysis of 1 independent variable at 6 levels where the total degrees of freedom is 24, what is the treatment degrees of freedom?

- 23 (distractor)
- 5 ($a - 1, 6 - 1 = 5$)
- 20 (distractor)
- 3 (distractor, treatment df: $a - 1$)
- Not Enough Information

The Treatment d.f. is the number of levels within the experiment minus 1 ($a-1$), $6-1 = 5$

24. Match the following ANOVA Analysis degrees of Freedom to their correct equation:

Total Degrees of Freedom = $a*n - 1$

Treatment Degrees of Freedom = $a - 1$

Error Degrees of Freedom = $a(n-1)$

25. You're performing an ANOVA Analysis of 1 independent variable at 3 levels and you're measuring 10 samples per level. What is the total degrees of freedom?

- 2 (treatment d.f.)
- 27 (error d.f.)
- 9 (random distractor)
- 30 (random distractor)
- 29
- Not Enough Information

Recall that in ANOVA, the total degrees of freedom = $N - 1$, where $N = a*n - 1$, with a being the number of levels (treatment groups) and n being the number of replicates or samples per level (treatment group).

$$N = 3*10 - 1 = 29$$

26. You're performing an ANOVA Analysis of 1 independent variable at 7 levels where the total degrees of freedom is 35, what is the error degrees of freedom?

- 34
- 28
- 29
- 6
- 35
- 42
- Not Enough Information

First must solve for the Treatment D.F., which is equal to $7-1$, or 6. Then we can subtract 6 from 35 to get the error degrees of freedom of 29.

27. You're performing an ANOVA Analysis of 1 independent variable at 10 levels where the total degrees of freedom is 20, what is the treatment degrees of freedom?

- 20
- 10
- 9
- 30
- 11
- Not Enough Information

The Treatment D.F. is the number of levels within the experiment minus 1 ($a-1$), $10-1=9$

28. The one way ANOVA Analysis below has 10 treatment groups with the total degrees of freedom of 19.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)				
Error (Within)	55			
Total	100	19		

Calculate the Treatment Mean Square for this ANOVA Table.

- 4.5
- 5
- 5.5
- 6.1

First, we can solve for the treatment sum of squares by simply subtracting 55 from 100, to get a treatment sum of Square of 45.

Then we must solve for the degrees of freedom. For the treatment, the degrees of freedom is equal to the number of treatment levels (10) - 1; so 9 degrees of freedom. Then we can solve for the error degrees of freedom by subtracting 19 - 9; so 10 degrees of freedom.

Then we can calculate the mean squares for the treatment as the treatment sum of square (45) divided by the treatment degrees of freedom (9): $45/9 = 5$.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	45	9	5	0.90
Error (Within)	55	10	5.5	
Total	100	19		

29. The one way ANOVA Analysis below has 15 treatment groups with the total degrees of freedom of 45.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	30			
Error (Within)				
Total	45	45		

Calculate the Error Mean Square for this ANOVA Table.

- 1.76
- 2.14
- 1.07
- 0.88
- 2.43

First, we can solve for the error sum of squares by simply subtracting 30 from 45, to get a error sum of squares of 15.

Then we must solve for the degrees of freedom. For the treatment, the degrees of freedom is equal to the number of treatment levels (15) - 1; so 14 degrees of freedom. Then we can solve for the error degrees of freedom by subtracting 30 - 14; so 17 degrees of freedom.

Then we can calculate the error mean squares as the error sum of square (15) divided by the error degrees of freedom (17): $15/17 = 0.88$

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	30	14	2.14	2.43
Error (Within)	15	17	0.88	
Total	45	30		

30. The one way ANOVA Analysis below has 3 treatment groups with the total degrees of freedom of 50.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	20			
Error (Within)				
Total	100	50		

What is the decision for this ANOVA Analysis if we choose the alpha risk to be 10%?

- Fail to Reject the Null
- **Reject the Null**
- Not Enough Information

First, we can solve for the error sum of squares by simply subtracting 20 from 100, to get an error sum of square of 80.

Then we must solve for the degrees of freedom. For the treatment, the degrees of freedom is equal to the number of treatment levels (3) - 1; so 2 degrees of freedom. Then we can solve for the error degrees of freedom by subtracting 50 - 2; so 48 degrees of freedom.

Then we can calculate the mean squares of the error and treatment by dividing the sum of squares by the degrees of freedom, which are 10 and 1.67 respectively.

The F-value can then be calculated by taking the ratio of MST to MSE which is $10 / 1.67 = 5.98$.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	20	2	10	5.98
Error (Within)	80	48	1.67	
Total	100	50		

Now to determine if this F-statistic is within our outside of the rejection region, we need to look up the critical F-value from the [NIST Table](#).

Recall that ANOVA is considered a two-sided test, so we're looking for the upper critical value of the F Distribution at the intersection of 2 degrees of freedom (v_1) and 48 degrees of freedom (v_2).

$$F_{\text{critical}} = F_{.05(2,48)} = 3.191$$

Since our F-statistic (5.98) is greater than F_{critical} (3.191), we must reject the null hypothesis.

31. The one way ANOVA Analysis below has 7 treatment groups with the total degrees of freedom of 38.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)				
Error (Within)	66			
Total	84	38		

What is the decision for this ANOVA Analysis if we use choose alpha risk to be 10%?

- Fail to Reject the Null
- Reject the Null
- Not Enough Information

First, we can solve for the treatment sum of squares by simply subtracting 66 from 84, to get a treatment sum of square of 18.

Then we must solve for the degrees of freedom. For the treatment, the degrees of freedom is equal to the number of treatment levels (7) - 1; so 6 degrees of freedom. Then we can solve for the error degrees of freedom by subtracting 38 - 6; so 32 degrees of freedom.

Then we can calculate the mean squares of the error and treatment by dividing the sum of squares by the degrees of freedom, which are 3.00 and 2.06 respectively.

The F-value can then be calculated by taking the ratio of MST to MSE which is $3.00 / 2.06 = 1.45$.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	18	6	3.00	1.45
Error (Within)	66	32	2.06	
Total	84	38		

Now to determine if this F-statistic is within our outside of the rejection region, we need to look up the critical F-value from the [NIST Table](#).

Recall that ANOVA is considered a two-sided test, so we're looking for the upper critical value of the F Distribution at the intersection of 6 degrees of freedom (v_1) and 32 degrees of freedom (v_2).

$$F_{\text{critical}} = F_{.05(2,48)} = 2.399$$

Since our F-statistic (1.45) is less than F_{critical} (2.399), we must fail to reject the null hypothesis.