# Problem Set For Point Hypothesis Testing

1. **A ________________ is defined as a statistical process used to make a decision between two mutually exclusive hypothesis.**

A. Hypothesis Test
B. Confidence Interval
C. Regression Analysis
D. Designed Experiment
E. Probability Assessment

2. **Hypothesis testing makes use of the _______________ to assess the probability of the sample statistic and distinguish between the null and alternative hypothesis.**

- Sampling Distribution
- Central Limit Theorem
- Probability Distribution
- Confidence Interval
- Theory of Constraints

3. **Match the following terms with their appropriate location on this table (A - F) of Null & Alternative Hypothesis:**

- Two Tail Test
- **Le**ft Tail Test
- $\mu > \mu_0$
- $\mu < \mu_0$
- $\mu = \mu_0$
- $\mu \leq \mu_0$

| B | A | Right Tail Test |
|---|---|---|
| $H_0$: $\mu \geq \mu_0$ | $H_0$: E | $H_0$: F |
| $H_a$: D | $H_a$: $\mu \neq \mu_0$ | $H_a$: C |

4. **Identify all of the statements below regarding hypothesis testing that are true:**

- Hypothesis testing is based on the initial assumption that the alternative hypothesis is true.
- If the p-value is less than the level of significance (alpha risk, α), you must reject the null hypothesis.
- The Chi-Squared Distribution is symmetric so the critical rejection criteria for the two tails are the same.
- The power of a hypothesis test is the probability of correctly rejecting the null hypothesis ($H_0$) when the null is actually false.
- When performing a two-tailed hypothesis test, the alpha risk is equally distributed between the two tails of the sample distribution.

5. **Identify all of the statements below regarding hypothesis testing that are true:**

- Rejecting the null hypothesis is considered a "strong claim" and means that the sample data was significant enough to reject the starting assumption that the null hypothesis was true.
- This null hypothesis, called $H_0$, is always a statement about the value of a sample statistic.
- A type I error is generally considered the worse of the two error types.
- When the null hypothesis is actually true but the hypothesis test rejects it, this is known as a type II error.
- the significance level (α) of our hypothesis test is also equal to the probability of a type I error.

6. **Identify all of the statements below regarding hypothesis testing that are False:**

- P-value stands for probability value and it represents the probability of observing a more extreme test statistic than the one observed.
- we're only interested in the power of a hypothesis test when the null hypothesis is in fact true.
- The alternative hypothesis always contain the equals sign.
- Using the normal distribution and the Z-transformation for hypothesis testing can only be used sample size is less than 30.

7. **Identify all of the statements below regarding hypothesis testing that are False:**

- Power can be thought of as the probability of avoiding a Type II error.
- The probability of occurrence of a Type I error is defined as the Beta (β) risk.
- The alpha risk in hypothesis testing is analogous to the consumers risk in the world of acceptance sampling.
- Failing to reject the null hypothesis is analogous to proving that the null hypothesis is true.

8. **Identify all of the statements below regarding hypothesis testing that are true:**

- Using the t-distribution for hypothesis testing of the population mean is required if your sample size is less than 30, or the population standard deviation is unknown.
- When we're comparing a sample variance against the population variance we must use the F Distribution.
- A type I error occurs due to random chance.
- when testing two population variances against each other we must use the chi-squared distribution.
- the significance level (α) is required to determine the rejection criteria for our hypothesis test.

9. **Identify all of the statements below regarding hypothesis testing that are false:**

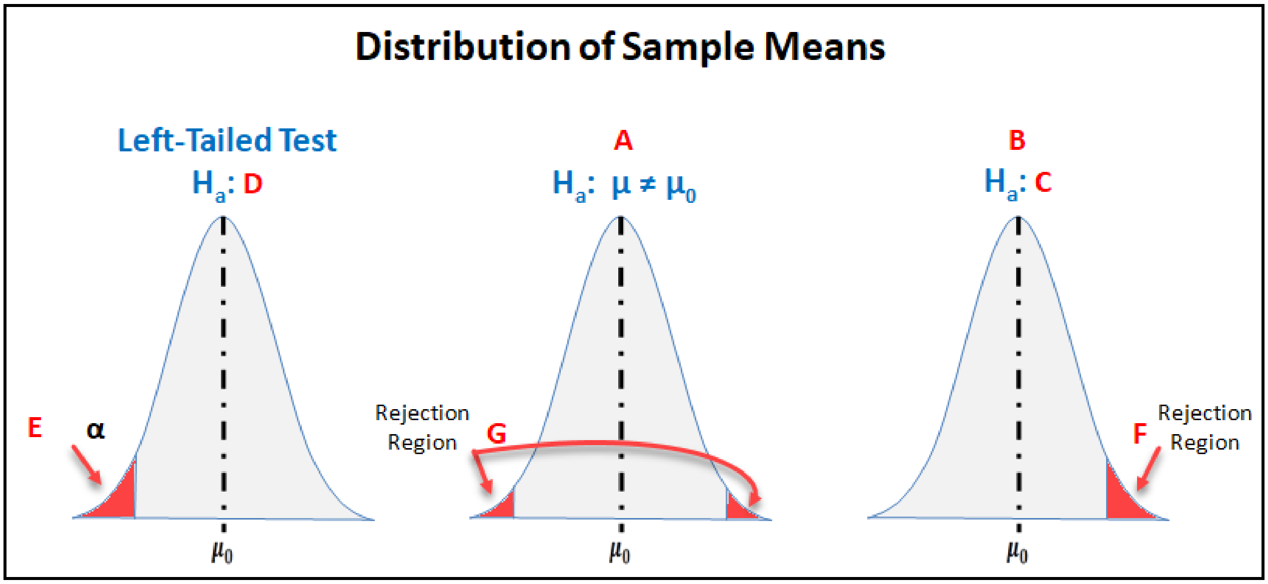
- The t-distribution can be safely used in population mean hypothesis test as an approximation of the Normal Distribution
- The alternate hypothesis is a statement about the sample statistic that is contradictory to the null hypothesis.
- A good tool to remember about the p-value is: If the p-value is low, the alternative hypothesis must go.
- A hypothesis test using the F-Test statistic can be used to confirm the assumption of equality (or homogeneity) of variances.
- The power of a hypothesis test is the probability of correctly accepting the null hypothesis when it is actually true.

10. **Below are 4 of the 6 steps associated with a Hypothesis Test. Put this in order with the first step on top and the last step on bottom:**

- Identify the Null Hypothesis ($H_0$) and the Alternative Hypothesis ($H_a$).
- Determine the Rejection Region for the Statistic of Interest (Mean, Variance, Proportion, etc).
- Calculate the Test Statistic from your Sample Data
- State the Decision in Terms of the Original Problem Statement

11. **Match the following terms with their appropriate location on this table of Null & Alternative Hypothesis:**

- Two Tail Test
- Right-Tail Test
- $\mu > \mu_0$
- $\mu < \mu_0$
- Rejection Region
- α
- α/2




12. **You're performing a hypothesis test for the population mean and you do not know the population standard deviation. You plan to sample 15 units from your population and you'd like to use a 1-sided test at a 10% significance level.**

**What is the rejection criteria for this hypothesis test?**

**Here's are the Z-Table & T-Tables for reference.**

- 1.285
- 1.341
- 1.345
- 1.650
- 1.753
- 1.761

13. **You're performing a hypothesis test for the population mean and you know the population standard deviation. You plan to sample 45 units from your population and you'd like to use a 2-sided test at a 5% significance level.**

    **What is the rejection criteria for this hypothesis test?**

    **Here's are the Z-Table & T-Tables for reference.**

- 1.341
- 1.345
- 1.650
- 1.761
- 1.960

14. **You're performing a hypothesis test for the population mean and you do not know the population standard deviation. You plan to sample 20 units from your population and you'd like to use a 2-sided test at a 5% significance level.**

    **What is the absolute value of the rejection criteria for this hypothesis test?**

    **Here's are the Z-Table & T-Tables for reference.**

- 1.650
- 1.729
- 1.725
- 1.960
- 2.093
- 2.086

15. **You're performing a hypothesis test for the population mean and you know the population standard deviation. You plan to sample 60 units from your population and you'd like to use a 1-sided test at a 1% significance level.**

    **What is the rejection criteria for this hypothesis test?**

    **Here's are the Z-Table & T-Tables for reference.**

- 1.650
- 1.96
- 2.33
- 2.39
- 2.58

**16. You're performing a hypothesis test for the population mean and your critical z-score is 1.65, and you've got a 2-sided test. If your z-statistic is -1.71, what would your conclusion be?**

- Accept the null hypothesis and thus reject the alternative hypothesis
- Fail to reject the null hypothesis
- Fail to reject the alternative hypothesis
- Reject the null hypothesis in favor of the alternative hypothesis

**17. You're performing a hypothesis test for the population mean critical t-value is 2.250, and you've got a 1-sided test. If your t-statistic is 2.200, what would your conclusion be?**

- Accept the null hypothesis and thus reject the alternative hypothesis
- Fail to reject the null hypothesis
- Fail to reject the alternative hypothesis
- Reject the null hypothesis in favor of the alternative hypothesis

**18. You're performing a hypothesis test for the population mean, and your sample mean is 10.5, your null hypothesis for the population mean is 11.5, your sample size is 30 and your population standard deviation is 2.**

**Calculate your z test statistic:**

- 2.738
- -2.738
- -0.5
- 0.5

**19. You're performing a hypothesis test for the population mean, and your sample mean is 228, your null hypothesis for the population mean is 246, your sample size is 10 and your sample standard deviation is 16.**

**Calculate your t test statistic:**

- 3.557
- -3.557
- -1.125
- -1.800

20. **You're performing a hypothesis test for the population mean, and your sample mean is 2.53, your null hypothesis for the population mean is 2.50, your sample size is 50 and your population standard deviation is 0.10.**

**Calculate your z test statistic:**

- 0.300
- 1.732
- 2.121
- 2.460

21. **You're performing a hypothesis test for the population mean, and your sample mean is 10.82, your null hypothesis for the population mean is 10.83, your sample size is 4 and your sample standard deviation is 0.05.**

**Calculate your t test statistic:**

- 0.300
- -0.200
- -0.300
- -0.400

22. **You manufacture a widget whose average length is 4.20 inches. You've upgraded your manufacturing equipment and you believe that it will not impact the overall length of the part.**

**You know the population standard deviation is 0.10 inches, and the sample mean of the 40 parts you measured is 4.24 inches. Using a 5% significance level to determine if the average length of the part has changed. Assume the length of the part is normally distributed.**

**Identify all of the statements below that are true:**

- The null hypothesis, $H_0$: $\mu = 4.24$ inches
- The alternative hypothesis, $H_a$: $\mu \neq 4.20$ inches
- The hypothesis test is a 1-sided test
- The critical rejection is $t_{crit} = 1.96$
- The test statistic is $z_{stat} = 2.53$
- The result of the test is the failure to reject the null hypothesis

**23. You manufacture a widget whose historical average tensile strength is 7000 PSI. You've changed the material composition of your widget and you believe that the tensile strength will improve.**

**You sample 10 parts and find a mean value of 7040 PSI and a sample standard deviation of 100 PSI. Using 10% significance level to determine if the average tensile strength has increased. Assume the tensile strength of the part is normally distributed.**

**Identify all of the statements below that are false:**

- The null hypothesis, $H_0$: $\mu \leq 7000$
- The alternative hypothesis, $H_a$: $\mu > 7{,}085$
- The hypothesis test is a 1-sided test
- The critical rejection is $t_{crit} = 1.372$
- The test statistic is $z_{stat} = 1.265$
- The end result of the test is the failure to reject the null hypothesis

**24. You're a surgeon and you believe that if your back surgery patients go to physical therapy four time a week (instead of 3 times), their recovery period will be shorter. Average recovery times for back surgery patients when they go 3 times a week is 9.6 weeks.**

**Determine the appropriate null and alternative hypothesis for this situation:**

- $\mu_0 = 9.6$ weeks
- $\mu_0 \leq 9.6$ weeks
- $\mu_0 \geq 9.6$ weeks
- $\mu_a \neq 9.6$ weeks
- $\mu_a < 9.6$ weeks
- $\mu_a > 9.6$ weeks

**25. You manufacture a widget whose average length is 4.20 inches. You've upgraded your manufacturing equipment and you believe that it will not impact the overall length of the part.**

**Determine the appropriate null and alternative hypothesis for this situation:**

- $\mu_0 = 4.20$ inches
- $\mu_0 \leq 4.20$ inches
- $\mu_0 \geq 4.20$ inches
- $\mu_a \neq 4.20$ inches
- $\mu_a < 4.20$ inches
- $\mu_a > 4.20$ inches

26. **You manufacture a part and the subsequently measure the length of the part to be 4.20 inches. Historically your measurement equipment has a variance of 0.25in$^2$. You're upgrading your measurement equipment which you believe will result in a reduction in variation.**

**Determine the appropriate null and alternative hypothesis for this situation:**

- $\sigma^2_0 = 0.25in^2$
- $\sigma^2_0 \leq 0.25in^2$
- $\sigma^2_0 \geq 0.25in^2$
- $\sigma^2_a \neq 0.25in^2$
- $\sigma^2_a < 0.25in^2$
- $\sigma^2_a > 0.25in^2$

27. **You manufacture a part and the subsequently measure the length of the part to be 4.20 inches.**

**Historically your measurement equipment has a variance of 0.25in$^2$. You're upgrading your measurement equipment which you believe will result in a reduction in variation.**

**You sample 15 parts and find a sample variance of 0.18in$^2$. Using a 5% significance level to determine if the variance has been reduced.**

**Identify all of the statements below that are True:**

- The null hypothesis, $H_0$: $\sigma^2 = 0.25in^2$
- The alternative hypothesis $H_a$: $\sigma^2 < 0.25in^2$
- The hypothesis test is a 2-sided test
- The critical rejection is $X^2_{crit} = 6.571$
- The test statistic is $X^2_{stat} = 10.80$
- The end result is the rejection of the null hypothesis in favor of the alternative hypothesis

28. **You manufacture a widget whose historical average tensile strength is 7000 PSI, and whose historical variance is 155 $PSI^2$.**

**You've changed the manufacturing process and you believe that the variance in the tensile strength will decrease.**

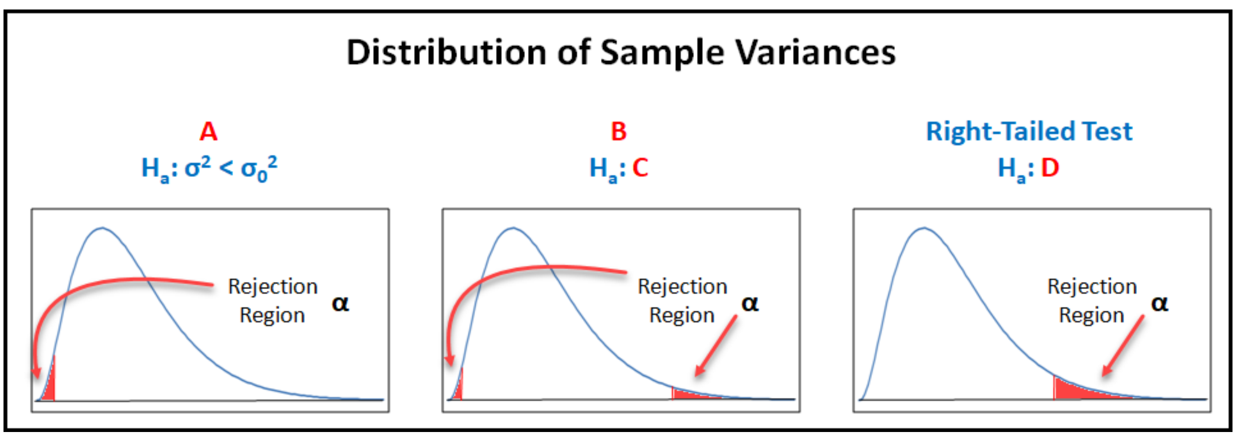
**You perform a hypothesis test at a 5% significance level to determine if the variance in tensile strength has been reduced. You sample 20 parts and find a sample standard deviation of 80 PSI.**

**Identify all of the statements below that are false:**

- The null hypothesis, $H_0$: $\sigma^2 \leq 155$ $PSI^2$
- The alternative hypothesis $H_a$: $\sigma^2 < 155$ $PSI^2$
- The hypothesis test is a 1-sided test
- The critical rejection is $X^2_{crit} = 8.907$
- The test statistic is $X^2_{stat} = 9.81$
- The end result is the rejection of the null hypothesis in favor of the alternative hypothesis
- At a 1% significance level, the result of the hypothesis test would be a rejection of the null hypothesis.

29. **Match the following terms with their appropriate location:**

- Left Tail Test
- Two-Tailed Test
- $\sigma^2 \neq \sigma_0^2$
- $\sigma^2 > \sigma_0^2$



**30. You're performing a hypothesis test for the population variance. You sample 20 parts and your sample variance is 2.53, your null hypothesis for the population variance is 2.50.**

**Calculate your chi-squared test statistic:**

- 17.79
- 18.22
- 19.23
- 20.24

**31. You're performing a hypothesis test for the population variance. You sample 5 parts and your sample variance is 700, your null hypothesis for the population variance is 850.**

**Calculate your chi-squared test statistic:**

- 3.29
- 4.11
- 4.85
- 6.07

**32. You're performing a hypothesis test for the population variance at a 10% significance level and a right-tailed test. You sample 20 parts, determine the critical chi-squared value (NIST Chi-Square Distribution Critical Value Table) for this test:**

- 27.204
- 28.412
- 30.144
- 31.410

**33. You're performing a hypothesis test for the population variance at a 1% significance level and a left-tailed test. You sample 10 parts, determine the critical chi-squared value (NIST Chi-Square Distribution Critical Value Table) for this test:**

- 1.152
- 1.479
- 2.088
- 2.558
- 2.700

34. **You're performing a hypothesis test for the population variance at a 5% significance level and a two-tailed test. You sample 15 parts, determine the critical chi-squared values (NIST Chi-Square Distribution Critical Value Table) for this test:**

- 5.629
- 6.262
- 6.571
- 7.261
- 23.685
- 24.996
- 26.119
- 27.488

35. **You're performing a hypothesis test for your population mean value and you want to increase the power of your test. Identify all of the statements below that would increase the power of your hypothesis test:**

- Increase Your Alpha Risk
- Decrease your sample size
- Decrease the variability in your population or process
- Having a smaller difference Between the hypothesized value and the actual parameter value
- Use a two-sided Hypothesis Test

36. **Match the following terms with their appropriate location on this table of Null & Alternative Hypothesis:**

- Correct Decision to Fail to Reject the Null Hypothesis
- Type II Error
- Type I Error
- Correct Decision to Reject the Null Hypothesis

| | | The Truth | |
|---|---|---|---|
| | | $H_0$ is True | $H_0$ is False |
| The Outcome of the Hypothesis Test | Fail to Reject $H_0$ | A | B |
| | Reject $H_0$ | C | *D* |

**37. Your manufacturing process has a particular step that is performed by two different machines that you believe to be identical. Prior to testing the mean values associated with your two processes, you want to test your assumption of the homogeneity of variances.**

**You take a sample of 10 units from machine A and 9 samples from Machine B and measure the sample variance of each machine to be $s_A^2$ = 0.12 in and $s_B^2$ = 0.20 in.**

**You want to use the significance level of 10% to test the hypothesis that the variances from each machine are equal.**
**(**critical F-Values using the NIST F-Table**) (**NIST Chi-Square Distribution Critical Value Table**)**

**Identify all of the statements below that are true:**

- The null hypothesis, $H_0$: $\sigma_a^2 \geq \sigma_b^2$
- The alternative hypothesis, $H_a$: $\sigma_a^2 \neq \sigma_b^2$
- The hypothesis test is a 1-sided test
- The critical rejection is defined by 0.309 < F-stat < 3.388
- The F-statistic is $f_{stat}$ = 0.60
- The end result of the test is the rejection of the null hypothesis in favor of the alternative hypothesis.

**38. You purchase the same component from two different vendors, and you want to assess if the variance from each vendor is the same.**

**You take a sample of 6 units from supplier A and 5 samples from supplier B and measure the sample variance for the tensile strength of each to be $s_A^2$ = 45 PSI$^2$ and $s_B^2$ = 110 PSI$^{2.}$**

**Using the significance level of 10%, test the hypothesis that the variances from these vendors are equal.**

**(**critical F-Values using the NIST F-Table**) (**NIST Chi-Square Distribution Critical Value Table**)**

**Identify all of the statements below that are false:**

- The null hypothesis, $H_0$: $\sigma_a^2 = \sigma_b^2$
- The alternative hypothesis, $H_a$: $\sigma_a^2 < \sigma_b^2$
- The hypothesis test is a two-sided test
- The critical rejection is defined by 5.192 < F-stat < 0.159
- The F-statistic is $f_{stat}$ = 2.44
- The end result of the test is the failure to reject the null hypothesis.

39. **Your manufacturing process has a historical yield loss average of 20%. You made a change to the process and you'd like to test if this change has resulted in a reduction of the yield loss. You sample 200 units and find 24 units are non-conforming.**

**Based on this sample, can we conclude that the proportion of defects in this lot is less than the historical average using a significance level of 5%.**

**Identify all of the statements below that are false:**

- The null hypothesis, $H_0$: $p \leq 20\%$
- The alternative hypothesis, $H_a$: $p < 20\%$
- The hypothesis test is a two-sided test
- The critical rejection is $z_{crit} = -1.96$
- The Z-statistic for the sample proportion is $Z_{stat} = -2.83$
- The end result of the test is the failure to reject the null hypothesis.

40. **Your manufacturing process has a historical yield loss average of 8%. You made a change to the process and you'd like to determine if this change has impacted the yield loss. You sample 50 units and find 8 units are non-conforming.**

**Based on this sample, can we conclude that the proportion of defects in this lot is different than the historical average using a significance level of 5%.**

**Identify all of the statements below that are True:**

- The null hypothesis, $H_0$: $p = 8\%$
- The alternative hypothesis, $H_a$: $p > 8\%$
- The hypothesis test is a two-sided test
- The critical rejection is $z_{crit} = -1.65$ & $Z_{crit} = 1.65$
- The Z-statistic for the sample proportion is $Z_{stat} = 2.83$
- The end result of the test is the rejection of the null hypothesis, in favor of the alternative hypothesis.

# Problem Set Solution:

1. A hypothesis test is defined as a statistical process used to make a decision between two mutually exclusive hypothesis.

2. Hypothesis testing makes use of the sampling distribution to assess the probability of the sample statistic and distinguish between the null and alternative hypothesis.

3. Match the following terms with their appropriate location on this table of Null & Alternative Hypothesis.

**A -** Two Tail Test
**B -** Left Tail Test
**C -** $\mu > \mu_0$
**D -** $\mu < \mu_0$
**E -** $\mu = \mu_0$
**F -** $\mu \leq \mu_0$

| B | A | Right Tail Test |
|---|---|---|
| $H_0$: $\mu \geq \mu_0$ | $H_0$: E | $H_0$: F |
| $H_a$: D | $H_a$: $\mu \neq \mu_0$ | $H_a$: C |

| Left Tail Test | Two Tail Test | Right Tail Test |
|---|---|---|
| $H_0$: $\mu \geq \mu_0$ | $H_0$: $\mu = \mu_0$ | $H_0$: $\mu \leq \mu_0$ |
| $H_a$: $\mu < \mu_0$ | $H_a$: $\mu \neq \mu_0$ | $H_a$: $\mu > \mu_0$ |

4. **Identify all of the statements below regarding hypothesis testing that are true:**

- Hypothesis testing is based on the initial assumption that the ~~**alternative**~~ **null** hypothesis is true.
- If the p-value is less than the level of significance (alpha risk, α), you must reject the null hypothesis. **(true)**
- The Chi-Squared Distribution is **NOT symmetric** so the critical rejection criteria for the two tails are the same.
- The power of a hypothesis test is the probability of correctly rejecting the null hypothesis ($H_0$) when the null is actually false. **(true)**
- When performing a two-tailed hypothesis test, the alpha risk is equally distributed between the two tails of the sample distribution. **(true)**

5. **Identify all of the statements below regarding hypothesis testing that are true:**

- Rejecting the null hypothesis is considered a "strong claim" and means that the sample data was significant enough to reject the starting assumption that the null hypothesis was true. **(true)**
- This null hypothesis, called $H_0$, is always a statement about the value of a ~~**sample statistic**~~ **population parameter**.
- A type I error is generally considered the worse of the two error types. **(true)**
- When the null hypothesis is actually true but the hypothesis test rejects it, this is known as a ~~**type II error**~~ **type 1 error**.
- the significance level (α) of our hypothesis test is also equal to the probability of a type I error. **(true)**

6. **Identify all of the statements below regarding hypothesis testing that are False:**

- P-value stands for probability value and it represents the probability of observing a more extreme test statistic than the one observed. **(true)**
- we're only interested in the power of a hypothesis test when the null hypothesis is in fact ~~**true**~~ **false**.
- The ~~**alternative**~~ **null** hypothesis always contain the equals sign.
- Using the normal distribution and the Z-transformation for hypothesis testing can only be used sample size is ~~**less than**~~ **greater than** 30.

**7. Identify all of the statements below regarding hypothesis testing that are False:**

- Power can be thought of as the probability of avoiding a Type II error. **(true)**
- The probability of occurrence of a Type I error is defined as the ~~**Beta (β)**~~ **alpha (α)** risk.
- The alpha risk in hypothesis testing is analogous to the ~~**consumers**~~ **producers** risk in the world of acceptance sampling.
- Failing to reject the null hypothesis is analogous to **proving that the null hypothesis is true** - ***this is an incorrect interpretation of a rejection of the null hypothesis.***

**8. Identify all of the statements below regarding hypothesis testing that are true:**

- Using the t-distribution for hypothesis testing of the population mean is required if your sample size is less than 30, or the population standard deviation is unknown. **(true)**
- When we're comparing a sample variance against the population variance we must use the ~~**F Distribution**~~ **Chi-Squared Distribution**.
- A type I error occurs due to random chance. **(true)**
- when testing two population variances against each other we must use the ~~**chi-squared distribution**~~ **F Distribution**.
- the significance level (α) is required to determine the rejection criteria for our hypothesis test. **(true)**

**9. Identify all of the statements below regarding hypothesis testing that are false:**

- The t-distribution can be safely used in population mean hypothesis test as an approximation of the Normal Distribution **(true)**
- The alternate hypothesis is a statement about the ~~**sample statistic**~~ **population parameter** that is contradictory to the null hypothesis.
- A good tool to remember about the p-value is: If the p-value is low, the ~~**alternative**~~ **null** hypothesis must go.
- A hypothesis test using the F-Test statistic can be used to confirm the assumption of equality (or homogeneity) of variances. **(true)**
- The power of a hypothesis test is the probability of correctly ~~**accepting**~~ **rejecting** the null hypothesis when it is actually ~~**true**~~ **false**.

**10. Below are 4 of the 6 steps associated with a Hypothesis Test. Put this in order with the first step on top and the last step on bottom:**

**Step 1.** Identify the **Null Hypothesis** ($H_0$) and the **Alternative Hypothesis** ($H_a$).

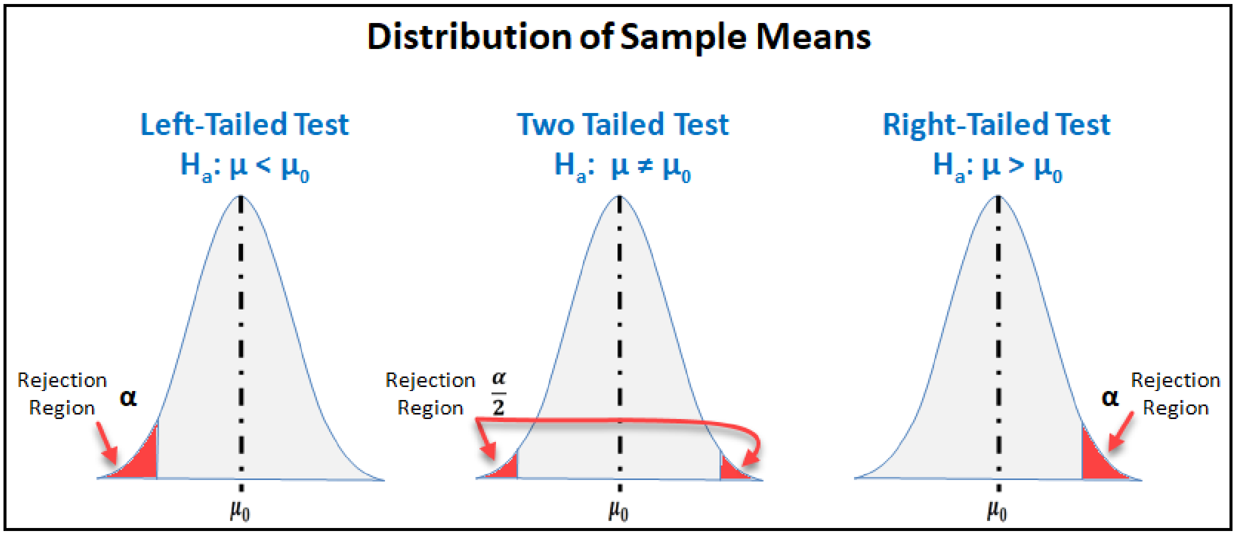
**Step 2.** Determine the **Rejection Region** for the Statistic of Interest (Mean, Variance, Proportion, etc).

**Step 3.** Calculate the **Test Statistic** from your Sample Data

**Step 4.** State the Decision in Terms of the **Original Problem Statement**

11. **Match the following terms with their appropriate location on this table of Null & Alternative Hypothesis:**

- A - Two Tail Test
- B - Right-Tail Test
- C - $\mu > \mu_0$
- D - $\mu < \mu_0$
- E- Rejection Region
- F - $\alpha$
- G -$\alpha/2$




12. **You're performing a hypothesis test for the population mean and you do not know the population standard deviation.**

    **You plan to sample 15 units from your population and you'd like to use a 1-sided test at a 10% significance level. Here's are the Z-Table & T-Tables for reference.**

    **What is the rejection criteria for this hypothesis test?**

*Because you do not know the population standard deviation, you must use the **t-distribution**, and not the normal distribution for your hypothesis test.*

*Sampling 15 units means you've got **14 degrees of freedom** and therefore the rejection criteria of a 1-sided test at the 10% significance level is* $\boldsymbol{t_{crit}}$ ***= 1.345***

**13. You're performing a hypothesis test for the population mean and you know the population standard deviation.**

**You plan to sample 45 units from your population and you'd like to use a 2-sided test at a 5% significance level. Here's are the Z-Table & T-Tables for reference.**

**What is the rejection criteria for this hypothesis test?**

*Because we know the population standard deviation and we're sampling more than 30 units we can use the normal distribution for your hypothesis test.*

*Based on the 2-sided test, and 5% significance level, we can look up the Z-value associated with 47.5% of the population, which is* $Z_{crit} = 1.960$

**14. You're performing a hypothesis test for the population mean and you do not know the population standard deviation.**

**You plan to sample 20 units from your population and you'd like to use a 2-sided test at a 5% significance level. Here's are the Z-Table & T-Tables for reference.**

**What is the absolute value of the rejection criteria for this hypothesis test?**

*Because you do not know the population standard deviation, you must use the* ***t-distribution****, and not the normal distribution for your hypothesis test.*

*Sampling 20 units means you've got* ***19 degrees of freedom*** *and therefore the rejection criteria of a 2-sided test at the 5% significance level is* $\boldsymbol{t_{crit} = 2.093}$

**15. You're performing a hypothesis test for the population mean and you know the population standard deviation.**

**You plan to sample 60 units from your population and you'd like to use a 1-sided test at a 1% significance level. Here's are the Z-Table & T-Tables for reference.**

**What is the rejection criteria for this hypothesis test?**

*Because we know the population standard deviation and we're sampling more than 30 units we can use the normal distribution for your hypothesis test.*

*Based on the 1-sided test, and 1% significance level, we can look up the Z-value associated with 49.0% of the population, which is* $Z_{crit} = \sim 2.33$

16. **You're performing a hypothesis test for the population mean and your critical z-score is 1.65, and you've got a 2-sided test. If your z-statistic is -1.71, what would your conclusion be?**

- Accept the null hypothesis and thus reject the alternative hypothesis
- Fail to reject the null hypothesis
- Fail to reject the alternative hypothesis
- **Reject the null hypothesis in favor of the alternative hypothesis**

*Because it is a 2-sided hypothesis test, you'll be looking for a value greater than 1.65, or less than -1.65. Because our value is less than -1.65, we can **reject the null hypothesis in favor of the alternative hypothesis.***

17. **You're performing a hypothesis test for the population mean critical t-value is 2.250, and you've got a 1-sided test. If your t-statistic is 2.200, what would your conclusion be?**

- Accept the null hypothesis and thus reject the alternative hypothesis
- Fail to reject the null hypothesis
- Fail to reject the alternative hypothesis
- Reject the null hypothesis in favor of the alternative hypothesis

*In this instance, our t-statistic (2.200) is less than our critical t-value (rejection criteria) of 2.250. Therefore we must **fail to reject the null hypothesis.***

**18. You're performing a hypothesis test for the population mean, and your sample mean is 10.5, your null hypothesis for the population mean is 11.5, your sample size is 30 and your population standard deviation is 2.**

**Calculate your z test statistic:**

*In this instance our hypothesis test sample size is greater than 30 and we know the population standard deviation; therefore we can use the normal distribution and z-score for our test statistic.*

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{10.5 - 11.5}{\frac{2}{\sqrt{30}}} = -2.738$$

**19. You're performing a hypothesis test for the population mean, and your sample mean is 228, your null hypothesis for the population mean is 246, your sample size is 10 and your sample standard deviation is 16.**

**Calculate your t test statistic:**

*In this instance our hypothesis test sample size is less than 30 and we only know the sample standard deviation, not the population standard deviation; therefore we must use the t-distribution and t-score for our test statistic.*

$$t - statistic = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{228 - 246}{\frac{16}{\sqrt{10}}} = -3.557$$

20. **You're performing a hypothesis test for the population mean, and your sample mean is 2.53, your null hypothesis for the population mean is 2.50, your sample size is 50 and your population standard deviation is 0.10.**

    **Calculate your z test statistic:**

*In this instance our hypothesis test sample size is greater than 30 and we know the population standard deviation; therefore we can use the normal distribution and z-score for our test statistic.*

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.53 - 2.50}{\frac{0.10}{\sqrt{50}}} = 2.121$$

21. **You're performing a hypothesis test for the population mean, and your sample mean is 10.82, your null hypothesis for the population mean is 10.83, your sample size is 4 and your sample standard deviation is 0.05.**

    **Calculate your t test statistic:**

*In this instance our hypothesis test sample size is less than 30 and we only know the sample standard deviation, not the population standard deviation; therefore we must use the t-distribution and t-score for our test statistic.*

$$t - statistic = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{228 - 246}{\frac{16}{\sqrt{10}}} = -0.400$$

22. **You manufacture a widget whose average length is 4.20 inches. You've upgraded your manufacturing equipment and you believe that it will not impact the overall length of the part.**

**You know the population standard deviation is 0.10 inches, and the sample mean of the 40 parts you measured is 4.24 inches. Using a 5% significance level to determine if the average length of the part has changed. Assume the length of the part is normally distributed.**

**Identify all of the statements below that are true:**

- The null hypothesis, $H_0$: μ = ~~4.24~~ 4.20 inches (False)
- The alternative hypothesis, $H_a$: μ ≠ 4.20 inches (True)
- The hypothesis test is a ~~1-sided~~ two sided test (False)
- The critical rejection is ~~$t_{crit}$~~ $z_{crit}$= 1.96 (False)
- The test statistic is $z_{stat}$ = 2.53 (True)
- The result of the test is the ~~failure to reject~~ rejection of the null hypothesis (False)

---

Because the problem statement is asking if the upgrade will "not impact the overall length", we can infer that this is a two-sided hypothesis test where the null and alternative hypothesis look like this:

**$H_0$: μ = 4.20 inches      $H_a$: μ ≠ 4.20 inches**

Because we know population standard deviation and we're sampling more than 30 units, we can use the normal distribution to determine the critical rejection region which can be found using the NIST Z-Table.

With a 2-sided test at 5% significance we're looking for the Z-score that captures 47.5% of the area of the distribution. This is at **Z = 1.96 and Z = -1.96**.

We can now calculate the Test Statistic from your Sample Data:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{4.24 - 4.20}{\frac{0.10}{\sqrt{40}}} = 2.53$$

Now we can compare the $z_{stat}$ (2.53) against the Rejection Criteria (-1.960, 1.960) and conclude that our test statistic is greater than our rejection criteria, therefore, we must reject the null hypothesis, in favor of the alternative hypothesis.

23. **You manufacture a widget whose historical average tensile strength is 7000 PSI. You've changed the material composition of your widget and you believe that the tensile strength will improve.**

    **You sample 10 parts and find a mean value of 7040 PSI and a sample standard deviation of 100 PSI. Using 10% significance level to determine if the average tensile strength has increased. Assume the tensile strength of the part is normally distributed.**

    **Identify all of the statements below that are false:**

    - The null hypothesis, $H_0$: $\mu \leq 7000$ (True)
    - The alternative hypothesis, $H_a$: $\mu >$ ~~7,085~~ 7,000 (False)
    - The hypothesis test is a 1-sided test (True)
    - The critical rejection is $t_{crit}$ = ~~1.372~~ 1.383 (False)
    - The test statistic is $z_{stat}$ = 1.265 (True)
    - The end result of the test is the failure to reject the null hypothesis (True)

---

Because the problem statement is inferring that the change will "improve the tensile strength", we can conclude that this is a one-sided, right tailed hypothesis test where the null and alternative hypothesis look like this: **$H_0$: $\mu \leq 7000$** **$H_a$: $\mu > 7{,}000$**

Because we only know the sample standard deviation, and our sample size is less than 30, we must use the t-distribution. The critical z-score can be found using the NIST Critical T Values.

At 9 degrees of freedom, and 10% significance level, we have a **Critical T values of 1.383**.

**Now we can calculate the Test Statistic from your Sample Data:**

$$t\ statistic = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{7040 - 7000}{\frac{100}{\sqrt{10}}} = 1.265$$

Let's compare our test statistic (1.265) against our rejection criteria (1.383). Because our test statistic is less than our rejection criteria, **we fail to reject the null hypothesis.**

24. **You're a surgeon and you believe that if your back surgery patients go to physical therapy four time a week (instead of 3 times), their recovery period will be shorter. Average recovery times for back surgery patients when they go 3 times a week is 9.6 weeks.**

**Determine the appropriate null and alternative hypothesis for this situation:**

- $\mu_0 = 9.6$ **weeks**
- $\mu_0 \leq 9.6$ **weeks**
- $\mu_0 \geq 9.6$ **weeks**
- $\mu_a \neq 9.6$ **weeks**
- $\mu_a < 9.6$ **weeks**
- $\mu_a > 9.6$ **weeks**

*Because the problem statement is inferring that the change in physical therapy will reduce the recovery period, we can conclude that this is a one-sided lower tail test where the null and alternative hypothesis look like this:* ***$H_0$: $\mu_0 \geq 9.6$ weeks*** ***$H_a$: $\mu_a < 9.6$ weeks***

25. **You manufacture a widget whose average length is 4.20 inches. You've upgraded your manufacturing equipment and you believe that it will not impact the overall length of the part.**

**Determine the appropriate null and alternative hypothesis for this situation:**

- $\mu_0 = 4.20$ **inches**
- $\mu_0 \leq 4.20$ **inches**
- $\mu_0 \geq 4.20$ **inches**
- $\mu_a \neq 4.20$ **inches**
- $\mu_a < 4.20$ **inches**
- $\mu_a > 4.20$ **inches**

*Because the problem statement is asking if the upgrade will "not impact the overall length", we can infer that this is a two-sided hypothesis test where the null and alternative hypothesis look like this:*

***$H_0$: $\mu = 4.20$ inches*** ***$H_a$: $\mu \neq 4.20$ inches***

**26. You manufacture a part and the subsequently measure the length of the part to be 4.20 inches. Historically your measurement equipment has a variance of $0.25in^2$. You're upgrading your measurement equipment which you believe will result in a reduction in variation.**

**Determine the appropriate null and alternative hypothesis for this situation:**

- $\sigma^2_0 = 0.25in^2$
- $\sigma^2_0 \leq 0.25in^2$
- $\sigma^2_0 \geq 0.25in^2$
- $\sigma^2_a \neq 0.25in^2$
- $\sigma^2_a < 0.25in^2$
- $\sigma^2_a > 0.25in^2$

*Because the problem statement is inferring that the change in equipment will reduce the variance, we can conclude that this is a one-sided lower tail test where the null and alternative hypothesis look like this:* ***$H_0$: $\sigma^2_0 \geq 0.25in^2$ $H_a$: $\sigma^2_a < 0.25in^2$***

**27. You manufacture a part and the subsequently measure the length of the part to be 4.20 inches.**

**Historically your measurement equipment has a variance of $0.25in^2$. You're upgrading your measurement equipment which you believe will result in a reduction in variation.**

**You sample 15 parts and find a sample variance of $0.18in^2$. Using a 5% significance level to determine if the variance has been reduced.**

**Identify all of the statements below that are True:**

- The null hypothesis, $H_0$: ~~$\sigma^2 = 0.25in^2$~~ $^2 \geq 0.25in^2$
- The alternative hypothesis $H_a$: $\sigma^2 < 0.25in^2$ (True)
- The hypothesis test is a ~~two-sided~~ one-sided test
- The critical rejection is $X^2_{crit} = 6.571$ (True)
- The test statistic is $X^2_{stat}$ = ~~10.80~~ 10.08
- The end result is the failure to reject ~~rejection of~~ the null hypothesis in favor of the alternative hypothesis

---

Because the problem statement is asking if the upgrade will "result in a reduction in variance", we can infer that this is a one-sided, left tailed hypothesis test where the null and alternative hypothesis look like this: **$H_0$: $\sigma^2 \geq 0.25in^2$** **$H_a$: $\sigma^2 < 0.25in^2$**

Using the NIST Chi-Square Distribution Critical Value Table, we can look up the rejection criteria at 14 degrees of freedom and a 5% Probability, determining that the **lower-tail critical value of 6.571**.

**Now we can calculate the Test Statistic from your Sample Data:**

$$\textit{Chi Squared Test Statistic}: X^2 = \frac{(N-1)s^2}{\sigma^2} = \frac{(15-1)0.18}{0.25} = 10.08$$

Finally, we can compare our chi-squared test statistic (10.08) against the lower-tail critical value rejection criteria (**6.571**). Because our test statistic is not less than our rejection criteria, **we fail to reject the null hypothesis.**

28. **You manufacture a widget whose historical average tensile strength is 7000 PSI, and whose historical variance is 155 PSI$^2$.**

    **You've changed the manufacturing process and you believe that the variance in the tensile strength will decrease.**

    **You perform a hypothesis test at a 5% significance level to determine if the variance in tensile strength has been reduced. You sample 20 parts and find a sample standard deviation of 80 PSI.**

    **Identify all of the statements below that are false:**

    - The null hypothesis, $H_0$: ~~$\sigma^2 \leq 155$ PSI$^2$~~ $\sigma^2 \geq 150$ PSI$^2$
    - The alternative hypothesis $H_a$: $\sigma^2 < 155$ PSI$^2$ (True)
    - The hypothesis test is a 1-sided test (True)
    - The critical rejection is $X^2_{crit}$ = ~~8.907~~ 9.81
    - The test statistic is $X^2_{stat}$ = 9.81 (True)
    - The end result is the rejection of the null hypothesis in favor of the alternative hypothesis (True)
    - At a 1% significance level, the result of the hypothesis test would be a failure to reject ~~rejection of~~ the null hypothesis.

---

Because the problem statement is asking if the change "result in a decrease in variance", we can infer that this is a one-sided, left tailed hypothesis test where the null and alternative hypothesis look like this: **$H_0$: $\sigma^2 \geq 150$ PSI$^2$** **$H_a$: $\sigma^2 < 150$ PSI$^2$**

Using the NIST Chi-Square Distribution Critical Value Table, we can look up the rejection criteria at 19 degrees of freedom and a 5% Probability, determining that the **lower-tail critical value of 10.117.**

**Now we can calculate the Test Statistic from your Sample Data:**

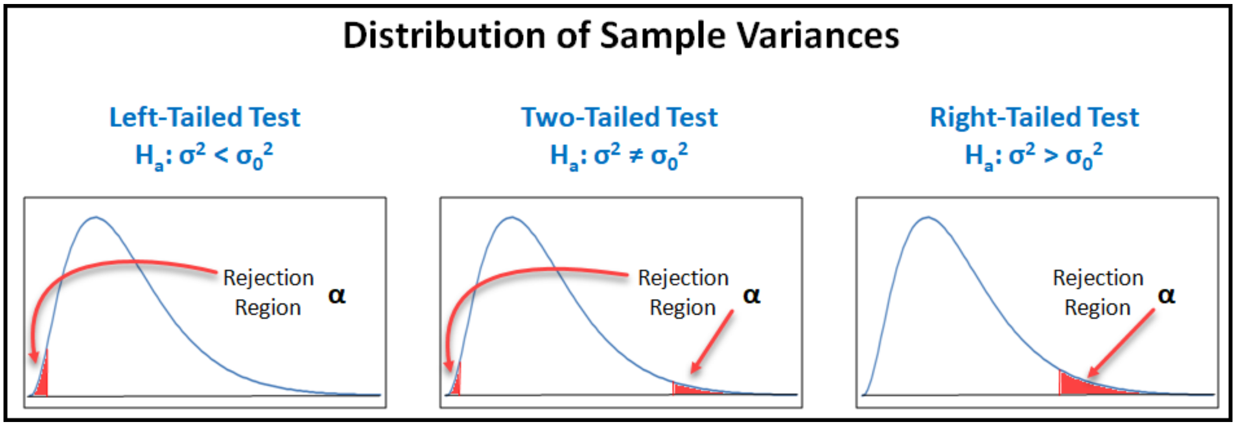
$$\textbf{Chi Squared Test Statistic}: X^2 = \frac{(N-1)s^2}{\sigma^2} = \frac{(20-1)80}{155} = 9.81$$

Finally, we can compare our chi-squared test statistic (**9.81**) against the lower-tail critical value rejection criteria (**10.117**). Because our test statistic is less than our rejection criteria, **we must reject the null hypothesis in favor of the alternative hypothesis.**

If we had chosen a 1% significance level, the rejection criteria would become 7.633, which means at that significance level we would **fail to reject the null hypothesis.**

**29. Match the following terms with their appropriate location:**

- A - Left Tail Test
- B - Two-Tailed Test
- C - $\sigma^2 \neq \sigma_0^2$
- D - $\sigma^2 > \sigma_0^2$




**30. You're performing a hypothesis test for the population variance. You sample 20 parts and your sample variance is 2.53, your null hypothesis for the population variance is 2.50.**

**Calculate your chi-squared test statistic:**

$$\boldsymbol{Chi\ Squared\ Test\ Statistic: X^2 = \frac{(N-1)s^2}{\sigma^2} = \frac{(20-1)2.53}{2.50} = 19.23}$$

**31. You're performing a hypothesis test for the population variance. You sample 5 parts and your sample variance is 700, your null hypothesis for the population variance is 850.**

**Calculate your chi-squared test statistic:**

$$\boldsymbol{Chi\ Squared\ Test\ Statistic: X^2 = \frac{(N-1)s^2}{\sigma^2} = \frac{(5-1)700}{850} = 3.29}$$

32. **You're performing a hypothesis test for the population variance at a 10% significance level and a right-tailed test. You sample 20 parts, determine the critical chi-squared value (NIST Chi-Square Distribution Critical Value Table) for this test:**

*Using the NIST Chi-Square Distribution Critical Value Table, we can look up the rejection criteria at 19 degrees of freedom and a 90% Probability to determine the* ***upper-tail (right tail) critical value of 27.207.***

33. **You're performing a hypothesis test for the population variance at a 1% significance level and a left-tailed test. You sample 10 parts, determine the critical chi-squared value (NIST Chi-Square Distribution Critical Value Table) for this test:**

*Using the NIST Chi-Square Distribution Critical Value Table, we can look up the rejection criteria at 9 degrees of freedom and a 1% Probability to determine the* ***lower-tail (left tail) critical value of 2.088.***

34. **You're performing a hypothesis test for the population variance at a 5% significance level and a two-tailed test. You sample 15 parts, determine the critical chi-squared values (NIST Chi-Square Distribution Critical Value Table) for this test:**

*Using the NIST Chi-Square Distribution Critical Value Table, we can look up the rejection criteria at 14 degrees of freedom at both the 97.5% Probability for the Upper/Right Tail and 2.5% Probability for the left/lower tail.*

***The lower-tail (left tail) critical value is 5.629***

***The upper-tail (right tail) critical value is 26.119***

**35. You're performing a hypothesis test for your population mean value and you want to increase the power of your test. Identify all of the statements below that would increase the power of your hypothesis test:**

- Increase Your Alpha Risk (True)
- ~~Decrease~~ Increase your sample size
- Decrease the variability in your population or process (True)
- Having a ~~smaller~~ larger difference Between the hypothesized value and the actual parameter value
- Use a ~~two-sided~~ one-sided Hypothesis Test

*You can increase the power of your test by increasing your alpha risk. This increases the likelihood or rejecting the null hypothesis. Additionally, decreasing the variability of your process or population increases the power of your hypothesis test.*

*Decreasing sample size will decrease the power of your test. Increasing the sample size improves the power of your hypothesis test.*

*Similarly, using a two-sided hypothesis test decreases the power of your hypothesis test as it spreads your alpha risk out between two tails and thus makes it less likely to reject the null hypothesis.*

*Additionally, having a smaller difference between the hypothesis value and the true value reduces the power of your test. You're more likely to reject the null hypothesis if the difference is larger.*

**36. Match the following terms with their appropriate location on this table of Null & Alternative Hypothesis:**

- A - Correct Decision to Fail to Reject the Null Hypothesis
- B - Type II Error
- C - Type I Error
- D - Correct Decision to Reject the Null Hypothesis

| | | The Truth | |
|---|---|---|---|
| | | $H_0$ is True | $H_0$ is False |
| The Outcome of the Hypothesis Test | Fail to Reject $H_0$ | Correct Decision | INCORRECT DECISION (Type II Error) Beta (β) Risk |
| | Reject $H_0$ | INCORRECT DECISION (Type I Error) Alpha (α) risk | Correct Decision *Power (1 - β)* |

**37. Your manufacturing process has a particular step that is performed by two different machines that you believe to be identical. Prior to testing the mean values associated with your two processes, you want to test your assumption of the homogeneity of variances.**

**You take a sample of 10 units from machine A and 9 samples from Machine B and measure the sample variance of each machine to be $s_A^2$ = 0.12 in and $s_B^2$ = 0.20 in.**

**You want to use the significance level of 10% to test the hypothesis that the variances from each machine are equal.**
**(**[critical F-Values using the NIST F-Table]**) (**[NIST Chi-Square Distribution Critical Value Table]**)**

**Identify all of the statements below that are true:**

- The null hypothesis, $H_0$: ~~$\sigma_a^2 \geq \sigma_b^2$~~ $\sigma_a^2 = \sigma_b^2$
- The alternative hypothesis, $H_a$: $\sigma_a^2 \neq \sigma_b^2$ (True)
- The hypothesis test is a ~~one-sided~~ two-sided test
- The critical rejection is defined by 0.309 < F-stat < 3.388 (True)
- The F-statistic is $f_{stat}$ = ~~0.60~~ 1.67
- The end result of the test is the failure to reject ~~rejection of~~ the null hypothesis in favor of the alternative hypothesis.

---

Since we're attempting to determine if these variances are different, here's what the null and alternative hypothesis look like. ***$H_0$: $\sigma_a^2 = \sigma_b^2$ & $H_a$: $\sigma_a^2 \neq \sigma_b^2$***

With machine B having a greater variance than machine A, we will consider $v_B$ to be $v_1$, and $v_A$ to be $v_2$.

**$v_1 = v_B = 9 - 1 = 8$ and $v_2 = v_A = 9$**

We can look up the [critical F-Values using the NIST F-Table] at the 10% alpha risk that's split between the upper and lower tail.

$$Upper\ Critical\ Value\ (Right\ Tail) = F_{\frac{\alpha}{2},(v_1,v_2)} = F_{0.05,(8,9)} = 3.388$$

$$Lower\ Critical\ Value\ (Left\ Tail) = F_{1-\frac{\alpha}{2},(v_1,v_2)} = \frac{1}{F_{\frac{\alpha}{2},(v_2,v_1)}} = \frac{1}{F_{0.05,(9,8)}} = \frac{1}{3.230} = 0.309$$

Now we can calculate the Test Statistic from our sample data:

$$F - Test\ Statistic: F = \frac{s_1^2}{s_2^2} = \frac{.20}{.12} = 1.67$$

Since our F-Statistic (1.67) is not greater than the upper critical value (3.388), nor is it less than the lower critical value (0.309), we fail to reject the null hypothesis.

**38. You purchase the same component from two different vendors, and you want to assess if the variance from each vendor is the same.**

**You take a sample of 6 units from supplier A and 5 samples from supplier B and measure the sample variance for the tensile strength of each to be $s_A^2$ = 45 $PSI^2$ and $s_B^2$ = 110 $PSI^2$.**

**Using the significance level of 10%, test the hypothesis that the variances from these vendors are equal.**

**Identify all of the statements below that are false:**

- The null hypothesis, $H_0$: $\sigma_a^2 = \sigma_b^2$ (True)
- The alternative hypothesis, $H_a$: ~~$\sigma_a^2 < \sigma_b^2$~~ $\sigma_a^2 \neq \sigma_b^2$
- The hypothesis test is a two-sided test (True)
- The critical rejection is defined by 5.192 < F-stat < 0.159 (True)
- The F-statistic is $f_{stat}$ = 2.44 (True)
- The end result of the test is the failure to reject the null hypothesis. (True)

---

Since we're attempting to determine if these variances are different, here's what the null and alternative hypothesis look like. ***$H_0$: $\sigma_a^2 = \sigma_b^2$ & $H_a$: $\sigma_a^2 \neq \sigma_b^2$***

With supplier B having a greater variance than supplier A, we will consider $v_B$ to be $v_1$, and $v_A$ to be $v_2$.

**$v_1 = v_B = 4$ and $v_2 = v_A = 5$**

We can look up the critical F-Values using the NIST F-Table at the 10% alpha risk that's split between the upper and lower tail.

$$Upper\ Critical\ Value\ \ (Right\ Tail) = F_{\frac{\alpha}{2},(v_1,v_2)} = F_{0.05,(4.5)} = \mathbf{5.192}$$

$$Lower\ Critical\ Value\ (Left\ Tail) = F_{1-\frac{\alpha}{2},(v_1,v_2)} = \frac{1}{F_{\frac{\alpha}{2},(v_2,v_1)}} = \frac{1}{F_{0.05,(5.4)}} = \frac{1}{6.256} = \mathbf{0.159}$$

Now we can calculate the Test Statistic from our sample data:

$$F - Test\ Statistic: F = \frac{s_1^2}{s_2^2} = \frac{110}{45} = 2.44$$

Since our F-Statistic (2.44) is not greater than the upper critical value (5.192), nor is it less than the lower critical value (0.159), **we must fail to reject the null hypothesis.**

**39. Your manufacturing process has a historical yield loss average of 20%. You made a change to the process and you'd like to test if this change has resulted in a reduction of the yield loss. You sample 200 units and find 24 units are non-conforming.**

**Based on this sample, can we conclude that the proportion of defects in this lot is less than the historical average using a significance level of 5%.**

**Identify all of the statements below that are false:**

- The null hypothesis, $H_0$: ~~p < 20%~~ p $\geq$ 20%
- The alternative hypothesis, $H_a$: p < 20% (True)
- The hypothesis test is a ~~two-sided~~ one-sided test
- The critical rejection is $z_{crit}$ = ~~-1.96~~ -1.65
- The Z-statistic for the sample proportion is $Z_{stat}$ = -2.83 (True)
- The end result of the test is the ~~failure to reject~~ rejection of the null hypothesis.

---

Based on the wording of the problem statement, this is a left-tail hypothesis test as we're attempting to determine if the population proportion is now less than the historical average of 20%.

**$H_0$: p $\geq$ 20% (0.20) & $H_a$: p < 20% (0.20)**

Based on our significance level of 5%, and the right-tail test, we can look up our critical Z-score that creates our rejection criteria using the NIST Tables of **$Z_{crit}$ = -1.65.**

Based on the problem statement we know that the hypothesized population proportion ($P_0$) is 0.20, and the sample size is 200:

$$p_O = 20\% \ (0.20) \ and \ n = 200$$

We can also calculate the sample proportion, p-hat:

$$\hat{p} = \frac{24}{200} = 12\% \ (0.12)$$

Next, we plug these into our z-transformation to calculate our test statistic:

$$Z_O = \frac{\hat{p} - p_O}{\sqrt{\frac{p_O(1-p_O)}{n}}} = \frac{0.12 - 0.20}{\sqrt{\frac{0.20(1-0.20)}{200}}} = -2.83$$

Because our test statistic ($Z_0$ = -2.83) is less than our rejection criteria ($Z_{crit}$ = -1.65), we can **reject our null hypothesis in favor of our alternative hypothesis** that the population proportion is less than 20%.

**40. Your manufacturing process has a historical yield loss average of 8%. You made a change to the process and you'd like to determine if this change has impacted the yield loss. You sample 50 units and find 8 units are non-conforming.**

**Based on this sample, can we conclude that the proportion of defects in this lot is different than the historical average using a significance level of 5%.**

**Identify all of the statements below that are True:**

- The null hypothesis, $H_0$: p = 8% (True)
- The alternative hypothesis, $H_a$: ~~p > 8%~~ p ≠ 8%
- The hypothesis test is a two-sided test (True)
- The critical rejection is $z_{crit}$ = ~~-1.65~~ -1.96 & $Z_{crit}$ = ~~1.65~~ 1.96
- The Z-statistic for the sample proportion is $Z_{stat}$ = ~~2.83~~ 2.085
- The end result of the test is the rejection of the null hypothesis, in favor of the alternative hypothesis. (True)

---

Based on the wording of the problem statement, this is a two-tail hypothesis test as we're attempting to determine if the proportion has changed from the historical average of 8%.

**$H_0$: p = 8% (0.08) & $H_a$: p ≠ 8% (0.08)**

Based on our significance level of 5%, and the two-tail test, we can look up our critical Z-score that creates our rejection criteria using the NIST Tables of **$Z_{crit}$ = -1.96 and $Z_{crit}$ = 1.96.**

Based on the problem statement we know that the hypothesized population proportion ($P_0$) is 0.20, and the sample size is 50:

$$p_O = 8\% \ (0.08) \ and \ n = 50$$

We can also calculate the sample proportion, p-hat:

$$\hat{p} = \frac{8}{50} = 16\% \ (0.16)$$

Next, we plug these into our z-transformation to calculate our test statistic:

$$Z_O = \frac{\hat{p} - p_O}{\sqrt{\frac{p_O(1-p_O)}{n}}} = \frac{.16 - .08}{\sqrt{\frac{0.08(1-0.08)}{50}}} = 2.085$$

Because our test statistic ($Z_0$ = 2.085) is greater than our rejection criteria ($Z_{crit}$ = 1.96), we can reject our null hypothesis in favor of our alternative hypothesis that the population proportion is not equal to 0.08%.